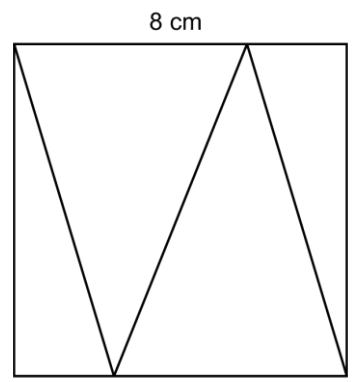
Mathematica Centrum

Together, let's shape the mathematicians of the future

NEWTON PREPARATORY TEST 2020
DETAILED SOLUTIONS

1. The sum of 9 and -7 is 2.

2. The number 21 is not prime because it has more than two divisors {1, 3, 7, 21}.



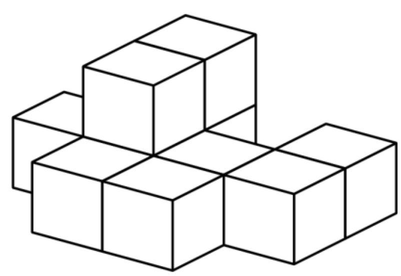
3. The result of (-4 + 8) - 4(5 - (-6)) is (4 - 4(11)) -40.

4. 100% x 100% + 50% x 200% = 1 x 1 + 0.5 x 2 = 1 + 1 = 2.

5. From n x -6 = -24, we find n = 4. The value of -n x -4 is (-4 x -4) 16.

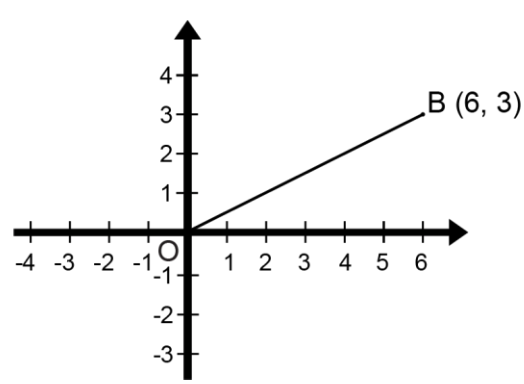
6. The area of the 4 triangles is (8 cm x 8 cm) 64 cm^2. The average area of the 4 triangles is (64 cm^2 ÷ 4) 16 cm^2.

7. The product of the digits of a 3-digit natural number cannot be equal to 38. The number 38 can only be written as the product of 2 x 19.



8. The factors of 8 are {1, 2, 4, 8}. The sum of all the factors of 8 is equal to (1 + 2 + 4 + 8) 15.

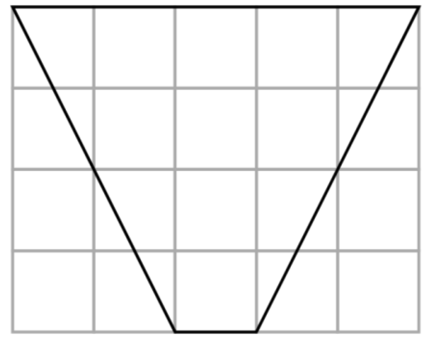
9. The 8 blocks on the bottom have (8 x 2) 16 glued faces. The two blocks on the top will add an extra 6 glued faces. Altogether, the 10 blocks have (16 + 6) 22 faces that are covered with glue.



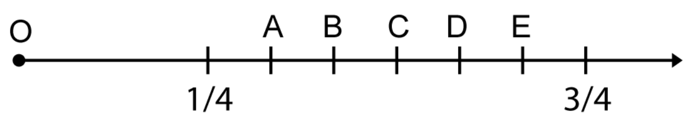
10. The coordinates of the image of point B after the rotation are (-3, 6). The opposite of the y coordinate of point B becomes the x coordinate of the image of point B and the x coordinate of point B becomes the y coordinate of its image.

11. Melissa uses 200 g of sugar for every 5 eggs. For 360 g of sugar she should use (5 ÷ 200 x 360) 9 eggs.

12. Every small square in the grid has an area of 1 cm^2. The area of the quadrilateral shown in the diagram is ((5 + 1) x 4 ÷ 2) 12 cm^2. You can also show that this quadrilateral has the same area as a 4 cm x 3 cm rectangle.



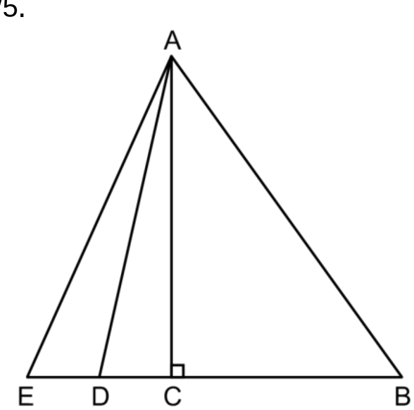
13. The fractions 1/4 and 3/4 are represented on the number line below. C is the midpoint between these two fractions. C represents the fraction 1/2 (50%). D is a point that lies 1/3 of the distance between 1/2 and 3/4. It is a point that represents the fraction (1/2 + 1/3 x 1/4) 7/12. This fraction is equal to 58 1/3%. Letter D represents the fraction whose value is closest to 55%.



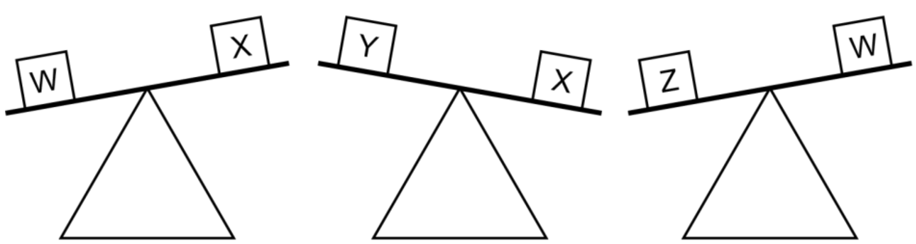
14. The two integers are -5 and 2. Their quotient could be -2/5.

15. 1 cm = 10 mm and 1 cm^2 = 10 mm x 10 mm = 100 mm^2. 10 cm^2 = 10 x 100 mm^2 = 1 000 mm^2.

16. If ED = DC = 1, then CB = 3ED = 3. The area of triangle ABE is (5 x AC ÷ 2). The area of triangle ACE is (2 x AC ÷ 2). The area of triangle ABE is 5/2 times (2.5 times) larger than the area of triangle ACE.

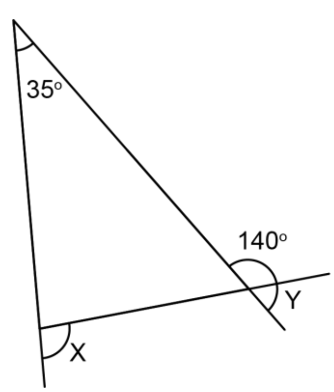


17. The diagram below tells us that W is heavier than X, that X is heavier than Y, and that Z is heavier than W. These diagrams can be represented by the following inequations: W > X, X > Y, and Z > W. Now let us

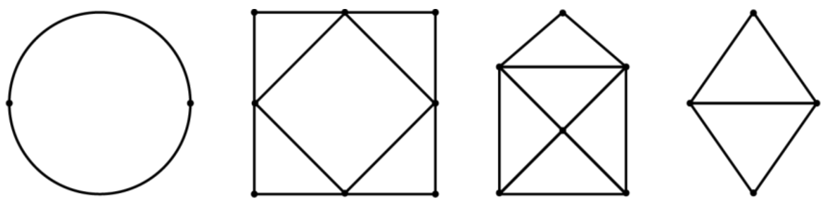


find out which inequation is false by using the transitive property. Transitivity stipulates that if W > X and X > Y, then W > Y. From the inequations Z > W and W > Y, we infer that Z > Y. The inequation Z > Y being true, we must conclude that Y > Z is false.

18. The value of Y is (180º - 140º) 40º. Find the values of all the remaining angles of the triangle (the sum of the angles of a triangle is always 180º). You will find that X = 75º and that the value of X + Y is (75º + 40º) 115º.



19. The points in a network are called vertices and the curves or straight lines are called arcs. The second network from the left has 8 vertices. The number of arcs meeting at a vertex is extremely important. If an even number of arcs meet at a vertex, we say that the vertex is even. If an odd number of arcs meet at a vertex, we say that the vertex is odd. As you can see, the second network has all even vertices. Four vertices have 2 arcs meeting at them and four vertices have 4 arcs meeting at them. Notice that the first network has two even vertices. The third network has 6 vertices, 4 even ones and 2 odd ones. The last network has 4 vertices, 2 even ones and 2 odd ones. If you can start at a certain vertex, follow a path through all the network and end at the same or another vertex in a way that each arc is traversed exactly once (you cannot go over the same arc more than once), we say that the network is traversable. A network that is traversable in a way that the starting vertex is the same as the stopping vertex is called a Euler circuit or Eulerian circuit. Go through the first network. Notice that you can start at any of the two vertices; go through all the network (never going over an arc more than once), you will end up at the same vertex where you started. Networks that have only even vertices are always Eulerian circuits. Also, verify that the second network is a Eulerian circuit. The other two networks are traversable networks but are not Eulerian circuits. They are called Euler paths. Euler paths start and end at different vertices. Verify that the last two networks are Euler paths.



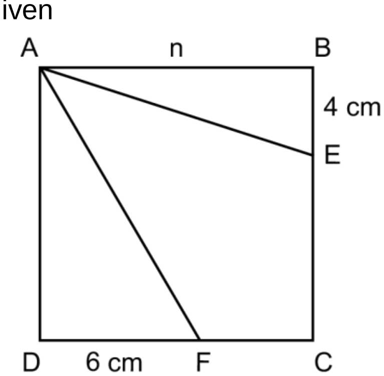
20. The LCM of 6 and 9 is 18. The GCD of 6 and 9 is 3. The product of the LCM and the GCD of 6 and 9 is (18 x 3) 54.

21. The average of all multiples of 7 between 0 and N is 52.5. The average of any regularly spaced sequence (like the sequence of all the multiples of 7 between 0 and N) is always given by the term in the middle of the sequence if it has an odd number of terms. In our sequence this term must be a multiple of 7. The average (52.5) is not a multiple of 7 and therefore we can infer that the sequence has an even number of terms. This average can be found by using the first term (7) and the last term in the sequence. The average being 52.5, we find that the last term is ((7 + X) ÷ 2 = 52.5) 98. The value of N could be represented by any natural number from 99 to105. The value N = 106 cannot represent a possible value of N.

22. The 7 litres of the final mixture will have (0.1 x 5 + 0.14 x 2) 0.78 litre of cream. The percentage of cream in the final mixture is (0.78 ÷ 7 x 100) 11 1/7%.

23. The next term in the infinite sequence: 0, 1, 2, 5, 12, 29, 70, … is (2 x 70 + 29) 169.

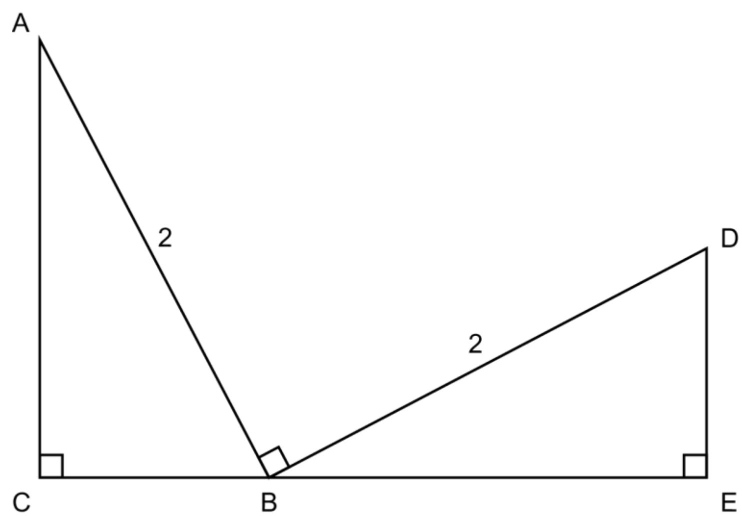
24. The area of square ABCD is n^2. The area of square ABCD is also given by n x 4 ÷ 2 + n x 6 ÷ 2 + 66. We can say that $n^2 = 2n + 3n + 66$. This equation can be written as $n^2 - 5n = 66$. The left side can be rewritten as the product of two algebraic expressions: n and n - 5. The equation can now be written as n x (n - 5) = 66. The factors of 66 are {1, 2, 3, 6, 11, 22, 33, 66}. We are looking for two factors n and n - 5 whose difference is 5 and whose product is 66. These two factors are 6 and 11. We find that n = 11 and n - 5 = 6. The value of n^2 is 121. The value of $2n^2$ is 242. The factors of 242 are {1, 2, 11, 22, 121, 242}. Of all the rectangles that have an area of 242 cm^2, the 11 cm x 22 cm rectangle is the one that has the smallest perimeter. Its perimeter is ((11 cm + 22 cm) x 2) 66 cm.



25. The congruency 18 ≡ 25 mod 7 tells us that 18 and 25 are congruent when they are divided by 7 (the modulo) because they leave the same remainder of 4. The congruency 17 ≡ 7 mod 10 has a remainder of 7. The congruency 7 ≡ 21 mod 7 has a remainder of 0, and the congruency 5 ≡ 17 mod 12 has a remainder of 5. The congruency 8 ≡ 15 mod 7 is the one with a remainder of 1.

26. Half of the students in a class are 12 years old or less and one sixth are 13 years old or more. The fraction of students that are between 12 and 13 years old is (1 - (1/2 + 1/6)) 1/3. The fraction of students that are between 12 and 13 years old (1/3) is 6 more than the fraction of students that are 13 years old or more (1/6). It follows that (1/3 - 1/6) 1/6 of the students is equal to 6 and 6/6 of the students is equal to 36. The number of students in the class that are between 12 and 13 years old is (1/3 x 36) 12.

27. The value of angle A + the value of angle ABC = 90º. The value of angle ABC + the value of angle DBE = 90º. This means that the value of angle A is equal to the value of angle DBE. It follows that the value of angle ABC is equal to the value of angle D. ΔABC ~ ΔBDE because their angles are equal. Not only are these two triangles similar, they are also congruent because the constant of proportionality K is equal to (2 ÷ 2) 1. Now, if CB = X, CA = 2X. From equation $X^2 + 4X^2 = 2^2$, we find that X = CB = 2/√5 and BE = CA = 2X = 4/√5.



28. He drove 40% of the distance (0.4 x 300 = 120 km) at a speed of 80 km/h and the rest of the distance (180 km) at a speed of 100 km/h. It took him (120 km ÷ 80 km/h + 180 km ÷ 100 km/h) 3.3 hours to complete the whole trip. Mathusalem’s average speed was (300 km ÷ 3.3h) 90.91 km/h. The speed which is closest to this average speed is 91 km/h.

29. The algebraic expression $(1/2 + X/2)^2 = (1/2 + X/2)(1/2 + X/2)$ becomes $1/4 + X/4 + X/4 + X^2/4$. The equation $1/4 + X/2 + X^2/4 + X^2 = 1$ becomes $5X^2 + 2X - 3 = 0$. We can write this equation as $(5X - 3)(X + 1) = 0$. We find X = 0.6 and X = -1. The value X = -1 must be discarded because X represents a length. A length cannot be negative. The value of X that verifies the equation $(1/2 + X/2)^2 + X^2 = 1^2$ is 0.6.