Mathematica Centrum

Together, let's shape the mathematicians of the future

LAGRANGE PREPARATORY TEST 2018
DETAILED SOLUTIONS

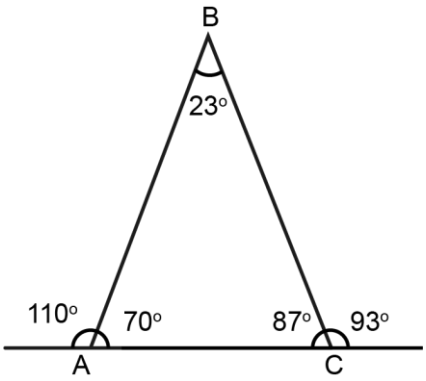
1. The numbers 3 (3 + 1 = 4), 15 (15 + 1 = 16), and 48 (1 + 48 = 49) yield a perfect square when you add 1 to them.

2. The largest possible sum, less than 10, of two consecutive prime numbers is (3 + 5) 8.

3. If 3/4 of a number is equal to 8, then 9/4 of the same number is equal to (8 x 3) 24.

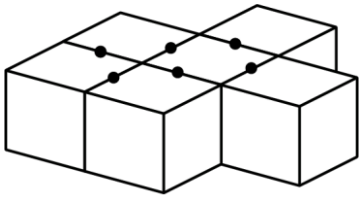
4. n ÷ 1/6 = 18 is equivalent to n x 6 = 18. The value of n x 2 is (3 x 2) 6.

5. The value of angle B is 23º.



6. (50% of 50%)% is equal to (1/2 x 1/2 = 1/4) ¼ % or 1/400.

7. Starting with -9, all integers are written in increasing order: -9, -8, -7, -6, -5, -4, -3, -2, … . The 20th digit that will be written is a (-9, -8, -7, …-1, 0, 1, 2, 3, … 9, **1**0) 1.



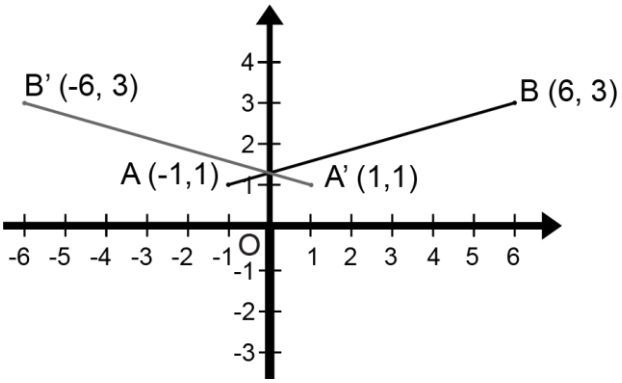
8. When the bottom layer of blocks are glued together, 12 faces will have glue on them. Another six faces will have glue on them when the two top blocks are put into place. In all, 18 faces of the eight blocks have glue on them.

P	Q	P X Q
0	12	0
1	11	11
2	10	20
3	9	27
4	8	32
5	7	35
6	6	36

9. The price after the 40% increase will be ($100 + $40) $140. The price after the 30% decrease will be ($150 - $45) $105. When bought together, the price of the two items will be ($140 + $105) $245. The price of the two items decreases by ($250 - $245 = $5 and ($5/ $250) x 100 = 2/100) 2%

10. If P + Q = 12, the greatest possible value of the expression P x Q is 36.

11. Line segment AB is reflected in the y-axis. The coordinates of the images of points A and B, after the reflection, are respectively, (1, 1) and (-6, 3).

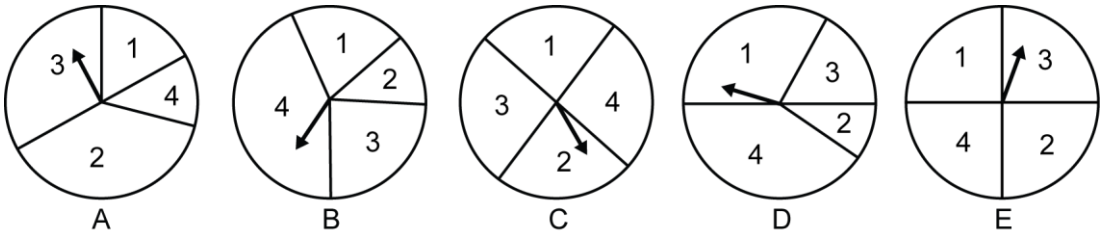


12. The result of the number of faces of a cube (6) plus the number of edges of a cube (12) plus the number of vertices of a cube (8) minus the number of angles of a cube (24) is equal to 2.

13. The LCM (3, 4, 5) = 60.

14. 1 cm^2 = 100 mm^2 and 20 cm^2 = 20 x 100 mm^2 = 2 000 mm^2

15. The probability of getting a 2 or a 4 with spinner B is more than 1/2.



16. There are 16 (1 x 1 squares), 9 (2 x 2 squares), 4 (3 x 3 squares), and 1 (4 x 4 square) in a 4 x 4 square (fig.1). In all, there are (16 + 9 + 4 + 1) 30 squares. In other words, there are ($4^2 + 3^2 + 2^2 + 1^2$) distinct squares in a 4 x 4 square. In a 3 x 3 square, you can count a total of ($3^2 + 2^2 + 1^2$) 13 distinct squares. Verify this assertion by counting the number of squares in the 3 x 3 square shown in the diagram (fig.2).

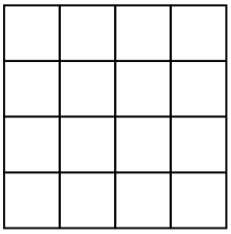


fig. 1

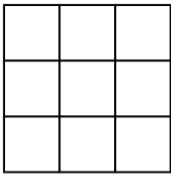


fig. 2

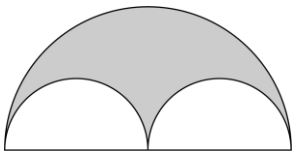
17. The area of the small triangle can be written as (b x h)/2. The area of the large triangle can be written as (2b x 2h)/2. The area of the small triangle is ((b x h)/2) ÷ 2/(2b x 2h) the area of the large triangle. The area of the small triangle is 1/4 of the area of the large triangle.

18. The sum of angle A + angle B + angle C = 180º.
Angle C + 20º + angle C + 10º + angle C = 180º.
We find that angle C = 50º and angle B = 60º.
The sum of angle B + angle C = 50º + 60º = 110º.

∠ A = ∠ C + 20º
∠ B = ∠ C + 10º
∠ C = ∠ C

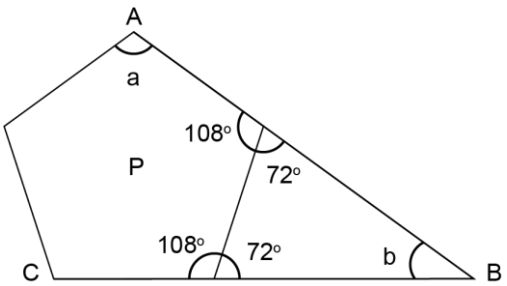
19. The only 3 prime numbers that add up to 19 are 3, 5, and 11. One of these prime numbers must be 11.

20. The area of the large half-circle is ($\pi 2^2 \div 2$) 2π.
The area of the two small half-circles is ($\pi 1^2$) π.
The area of the shaded surface is ($2\pi - \pi$) π.



21. There are 3 possibilities for the hundreds digit (1, 2, or 3), 3 for the tens digit, and 3 for the units digit. In all, there are 27 (3 x 3 x 3) 3-digit numbers that can be formed using the digits 1, 2, and 3.

22. Each angle of a regular pentagon is equal to (3 x 180º ÷ 5) 108º. The value of a is 108º. The value of b is (180º - 2 x 72º) 36º. The value of 2a + b is equal to (2 x 108º + 36º) 252º.



23. The area of the square is (4 x 4) 16. The area of the four circles is (4 x $\pi 1^2$) 4π. The area of the shaded surface is 16 - 4π.