Mathematica Centrum

Together, let's shape the mathematicians of the future

PYTHAGORAS PREPARATORY TEST 2016
DETAILED SOLUTIONS

1. The base of a pyramid has 6 sides. In all, this pyramid has (6 + 1) 7 vertices.

2. The expression 400 < 398 is false.

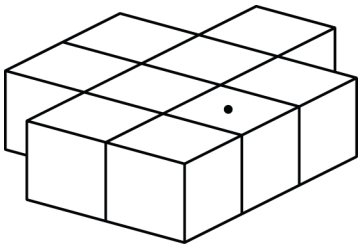
3. The difference between (7 x 12) 84 and (72 ÷ 8) 9 is (84 - 9) 75.

4. A period of 8 weeks is equal to (8 x 7) 56 days. A period of (56 + 8) 64 days represents more than 63 days.

5. The ten's digit of (428 - 348) 80 is 8.

6. There are about (6 x 30) 180 days or a little less than (180 ÷ 7) 26 weeks in a period of 6 months. You will go to the gym approximately (26 x 5) 130 times over a period of 6 months.

7. Nine blocks have been glued together, as shown in the diagram. There is only 1 block (the one with a dot) that has exactly 3 faces that have glue on them.



8. Mathew is X years old and Mathilda Y years old. The sum of their ages is presently X + Y. Three years ago, the sum of their ages was X + Y - 6.

9. From 1 to 100 there are 100 natural numbers. If we take away all 1-digit natural numbers (1 to 9) and the only 3-digit natural number (100), there is a total of (100 - 10) 90 2-digit natural numbers.

X	9	13	7
3	**27**	39	21
4	36	**52**	**28**
10	90	130	70

10. The expression that yields a sum that is even is 12 + 14 + 55 + 33.

11. Mathilda rolls a dice 30 times. She should expect to get a 5 (30 ÷ 6) 5 times.

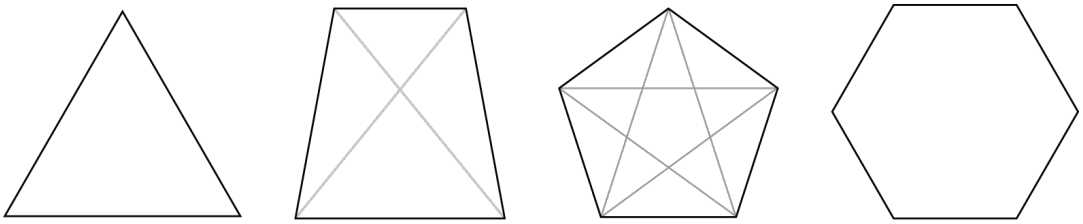
12. The number represented by a ?, that has a value closest to 30 is, 28.

13. 3 hundreds (300) + 50 ones + 16 tens (160) is equal to (300 + 50 + 160) 510.

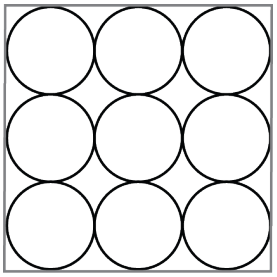
14. 2 m (200 cm) + 1 dm (10 cm) + 5 cm is equal to (200 + 10 + 5) 215 cm.

15. There are 3 different ways (10 x $2, 4 x $5, and (2 x $5 + 5 x $2)) to make change for a $20 bill if you were using $5 bills and $2 coins.

16. Zero diagonals can be drawn in a triangle. Two diagonals can be drawn in a quadrilateral, and 5 can be drawn in a pentagon. If you analyse closely these three numbers, you can see that they form a logical sequence. Indeed, 0 + **2** = 2, 2 + **3** = 5. The number of diagonals that can be drawn in a hexagon is (5 + **4**) 9.

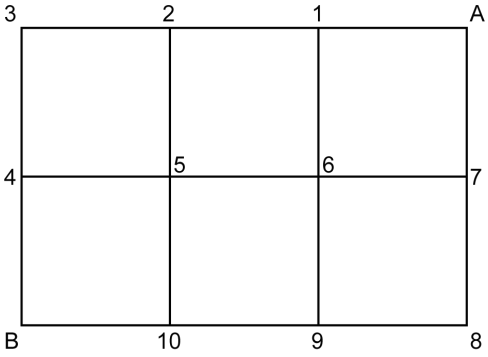


17. From N x N = 1 + 2 + 3 + 4 + 3 + 2 + 1, we deduct that N x N = 16 = 4 x 4 and that N = 4. The value of 10 x N is equal to (10 x 4) 40.



18. Andrea can stack 4 balls on the 9 balls that form the base. On these 4 balls, she can stack one other ball. She will need (4 + 1) 5 more balls to form this ''pyramid''.

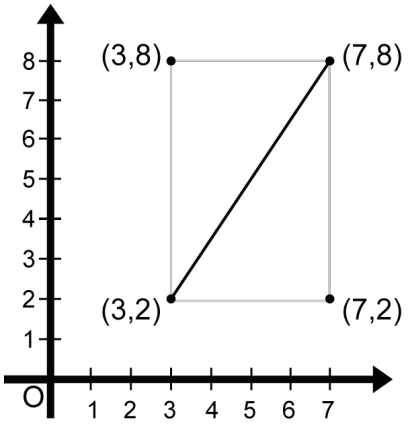
19. There are 10 different 500 m routes (A-1-2-3-4-B, A-1-2-5-4-B, A-1-2-5-10-B, A-1-6-5-4-B, A-1-6-5-10-B, A-1-6-9-10-B, A-7-6-5-4-B, A-7-6-5-10-B, A-7-6-9-10-B, and A-7-8-9-10-B) to get from point A to point B.



20. Melissa has bought 5¢ and 10¢ stamps for a total of 55¢. If she were to buy the same number of 5¢ stamps, but twice the number of 10¢ stamps, it would cost her $1.05. From these two premises, we can infer that the amount paid for the 10¢ stamps is (105¢ - 55¢) 50¢. The number of 10¢ stamps she has bought is (50¢ ÷10¢) 5 and that of 5¢ is ((55¢ - 50¢) ÷ 5¢) 1.

21. The divisors of 10 are {1, **2**, 5, 10}, those of 12 are {1, **2**, 3, 4, 6, 12}, and those of 36 are {1, **2**, 3, 4, 6, 9, 12, 18, 36}. The GCD of 10, 12, and 36 is 2.

22. Diagonal AB of a rectangle is represented in the diagram. The coordinates of one of the other two vertices of the rectangle are (7 , 2).



23. In the sequence: 1, 8, 15, 22, 29, … 113, each term is 7 more than the preceding term. We can say that 1 + 7 x ? = 113. The value of the ? is given by (113 - 1) ÷ 7. This value is 16. In this sequence, there are (16 + 1) 17 terms.

24. When he stops, after having covered 70% of the distance, Mathew has covered (70% x 30) 21 km. From the time he left his house to the time he got back, Mathew has travelled a distance of (21 x 2) 42 km.

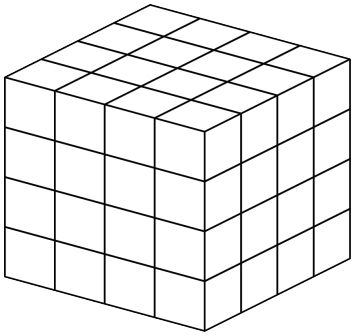
25. We can calculate the number of pages that Mathilda read by finding the numerator in the equation: 6/5 = ?/75. Mathilda has read (6 x 15) 90 pages.

26. The next number in the sequence: 1, 3, 4, 7, 11, … is (7 + 11) 18.

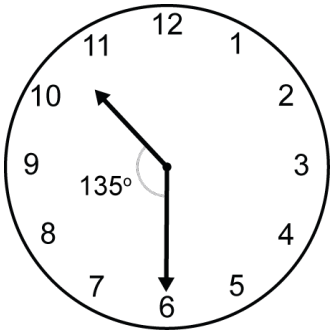
27. The difference between any two prime numbers is always even, except when one of the two prime numbers is the number 2 (the only even prime number). The second prime number is (5 + 2) 7. Their sum is (2 + 7) 9.

28. The value closest to the product of 999 x 1 000 x 1 001 is (1 000 x 1 000 x 1 000) 10^9.

29. Mathusalem is looking at large 4 x 4 x 4 cube from a certain angle (without moving and without moving the cube). How many of the small cubes are hidden (a cube is hidden if none of its faces can be seen)? The answer is 27. Look at the diagram. If you had only one cube (a 1 x 1 x 1 cube), how many cubes would be hidden? The answer is obviously 0. If you had a 2 x 2 x 2 cube, there would be (1^3) 1 small cube that would be hidden (look at the diagram and verify that 7 small cubes are visible). If you had a 3 x 3 x 3 cube, there would be 2^3 or 8 small cubes that would be hidden. Looking at a 4 x 4 x 4 cube, there are 3^3 or 27 hidden cubes.



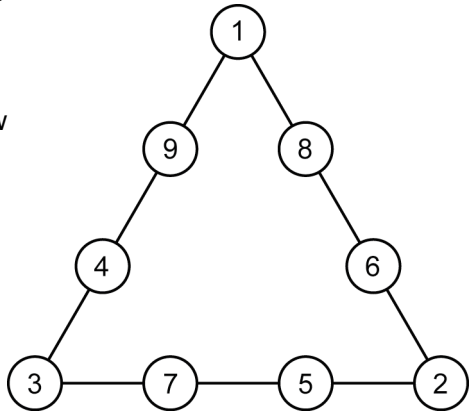
30. When the minute hand turns 360º, the hour hand turns (360º ÷ 12) 30º. When it is 10:30, the minute hand has turned 180º and the hour hand (180º ÷ 12) 15º. The measure of the angle formed by the hour and minute hands when it is 10:30 is (4 x 30º + 15º) 135º.



31. The largest fraction is 19/20.

32. Some prime numbers can be written as the sum of two square numbers: $5 = 1 + 4 = 1^2 + 2^2$, $13 = 4 + 9 = 2^2 + 3^2$, $17 = 1 + 16 = 1^2 + 4^2$. Let us write a few of the terms of the sequence of square numbers to help us identify the prime numbers that can be written this way. This sequence is 0, 1, 4, 9, 16, 25, 36, 49, … . 29 = 25 + 4, 61 = 36 + 25, 41 = 25 + 16, and 53 = 49 + 4. It is impossible to write the prime number 19 as the sum of two square numbers.

33. The sum of the digits from 1 to 9 is 45. We know that the sum of the 9 digits on the 3 sides must be (3 x 17) 51. This surplus of (51 - 45) 6 is due to the fact that the letters P, M, and N are counted twice. To get this sum of 17, the sum of P + M + N must be equal to 6. The diagram opposite represents one of the possible configurations of numbers that can be placed on each side of the triangle.



34. Mathew's car is travelling at 90 km/h, and Mathilda's at 100 km/h. Every hour, Mathilda will cover 10 km more than Mathew. Being only 2 km behind Mathew, it will take her (60 ÷5) 12 minutes to catch up to Mathew.