Mathematica Centrum

Together, let's shape the mathematicians of the future

NEWTON PREPARATORY TEST 2013

1. The cube root of the square of 8 is equal to

A) 3 B) 4 C) 16 D) 8 E) 2

2. The value of (-2 + 6) - (-6 + 2) is

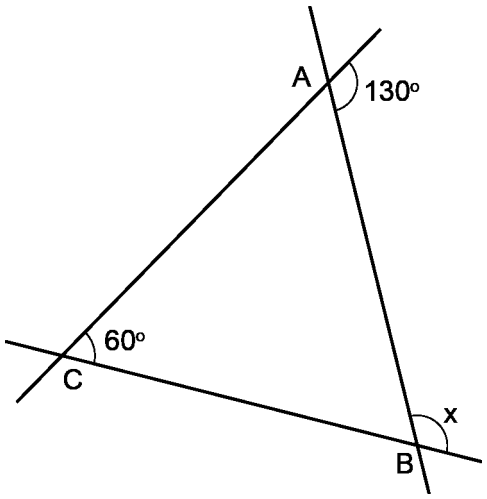
A) 4 B) 0 C) -8 D) 8 E) -4

3. 1/3 + 1/2 + 1/6 = ?

A) 1 B) 15/16 C) 7/8 D) 3/5 E) 5/6

4. What is the value of x in Δ ABC?

A) 65° B) 100° C) 110°
D) 70° E) 80°



5. The smallest prime factor of 105 is

A) 2 B) 15 C) 3
D) 5 E) 7

6. If n = √256 ÷ √81, what is the value of √n?

A) 3 B) 3/4 C) 2/3
D) 9/4 E) 4/3

7. The result of 3/5 x 2/3 x 5/4 is

A) 0.3 B) 0.4 C) 40% D) 0.5 E) 60%

8. 18% of 50 is equal to

A) 10% of 100 B) 9% of 100 C) 8% of 100 D) 5% of 200 E) 3% of 400

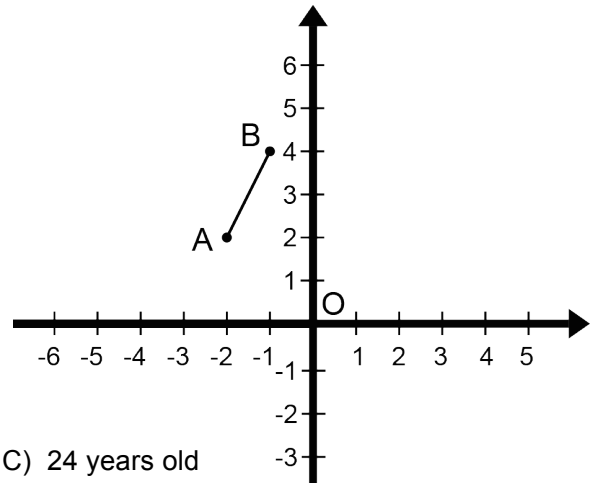
9. The number of minutes in 60 years is the same as the number of seconds in

A) one year B) 600 days C) 360 years D) 100 weeks E) 2 years

10. The result of $3^2 \times 5^2 + 3^2 \times 5^2 + 3^2 \times 5^2 + 3^2 \times 5^2$ is equal to

A) 10^2 B) 15^2 C) 30^2 D) 60^2 E) 32^2

11. What are the coordinates of the images of points A and B of line segment AB if it is turned 90º clockwise around point O?

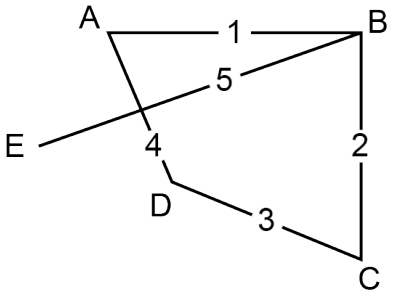


A) A'(2, -2), B'(4, -1)
B) A'(-4, 1), B'(-2, 2)
C) A'(2, 2), B'(4, 1)
D) A'(4, -1), B'(2, -2)
E) A'(4, 1), B'(2, 2)

12. Mathilda's age is one third that of Mathusalem's. If Mathusalem is 36 years older, what is Mathilda's age?

A) 14 years old B) 16 years old C) 24 years old
D) 18 years old E) 12 years old

13. Points A, B, C, D, and E represent five North American cities. The Nordair company wants to establish full air service between these cities. How many different air routes can it offer? (the diagram on the right shows 5 of the air routes it can offer)



A) 11 B) 15 C) 8
D) 9 E) 10

14. A 4-digit natural number is multiplied by a 2-digit natural number. The product could have

A) 6 digits B) 7 digits C) 4 digits D) 8 digits E) 9 digits

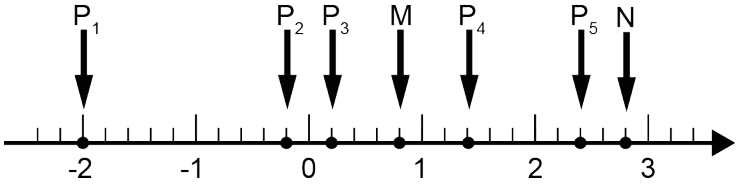
15. Which of the following numbers is not prime?

A) 3 B) 9 C) 53 D) 73 E) 13

16. The average of six numbers is 46. If two of these numbers are 46 and 34, what is the average of the other 4?

A) 52 B) 51 C) 49 D) 50 E) 48

17. Which point on the number line is 4 times further from point M than from point N?



A) P_1 B) P_3 C) P_2 D) P_5 E) P_4

18. The number which is a multiple of 6, but is not a multiple of 5 is

A) 45 B) 90 C) 180 D) 75 E) 186

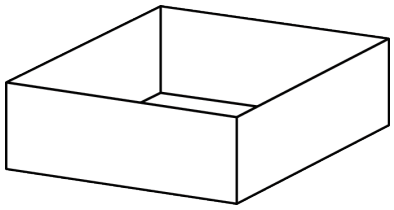
19. How many 5 cm x 8 cm sheets of paper can you get, if you cut 100 sheets of paper 11 cm x 17 cm?

A) 400 B) 150 C) 250 D) 600 E) 200

20. Four people (W, X, Y, and Z) are waiting in line at a banking machine. X is not second. Y is just behind X. Z is just in front of W who is neither first nor last. Who is third?

A) W B) Y C) Z D) X E) we cannot say

21. Using a 50 cm x 60 cm rectangular carton, Carol has made a box 9 cm high in which she can place Christmas balls. What area did she cut away from the carton in order to make this box?



A) 400 cm^2 B) 324 cm^2 C) 300 cm^2
D) 380 cm^2 E) 350 cm^2

22. Four blocks are placed on boards as shown in the diagram (fig.1). Mathew has to move the blocks so that they are stacked in the order shown in fig.2. He must move only one block at a time and he cannot place one block on another block whose number is smaller. Finally, he is allowed to put any block on an empty board. If the transfer of a block from one board to another counts as one move, what is the minimum number of moves that he must make in order to stack the blocks as shown in fig.2?

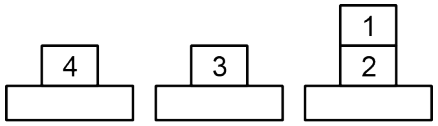


fig.1

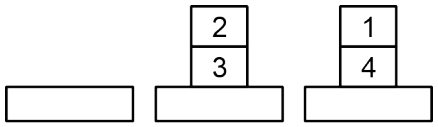


fig.2

A) 5 B) 4 C) 7
D) 6 E) 8

23. The product of all natural numbers from 1 to 12 (1 x 2 x 3 x … x 11 x 12) is equal to 12!. What is the maximum value of k for which 3^k is a factor of 12!?

A) 6 B) 4 C) 5 D) 3 E) 7

24. Melissa and Andrea go into a restaurant. There are 4 hooks on a wall. Each one hangs her hat on one of the hooks. How many different ways can they hang their hats?

A) 4 B) 12 C) 8 D) 24 E) 16

25. The natural number 3 113 is called a palindrome number. Reading from left to right or from right to left, it represents the same number. The numbers 44 and 717 are also palindrome numbers. How many palindrome numbers are there between 100 and 1 000?

A) 100 B) 9 C) 99 D) 180 E) 90

26. In mathematics, modular arithmetic (also called clock arithmetic) is simply an arithmetic for integers, where numbers start over after reaching a certain value – the modulus. There is one modular arithmetic that we are all familiar with – arithmetic modulo 12. Three hours after 11 o'clock, it is always 2 o'clock. We might think that 11 + 3 should give 14, but in a 12 hour clock, there is no hour called 14 o'clock, since the numbers start over after reaching the value 12 -- the modulus. This is called arithmetic modulo 12. We say that 14 o'clock and 2 o'clock are congruent, because they represent the same time. Mathematically, we represent this by writing $2 \equiv 14 \bmod 12$. We could also write that $0 \equiv 12 \bmod 12$ or that $5 \equiv 17 \bmod 12$. Note the use of the ≡ symbol (the symbol for congruency) and not the = symbol (the symbol for equality), because 2 is not equal to 14. These numbers are really congruent, because they represent the same time. Let us look at other examples. If today is Wednesday, what day will it be in 8 days, in 15 days? In 8 days, it will be ($8 \div 7 = 1$ R 1) a Thursday. In 15 days, it will also be a Thursday ($15 \div 7 = 2$ R 1). When 2 integers leave the same remainder, when they are divided by the same number, we say that they are congruent. The operation of finding the remainder is called the modulo operation. We can write that $8 \equiv 15 \bmod 7$. We can also write that $7 \equiv 21 \bmod 7$, $18 \equiv 25 \bmod 7$, and $17 \equiv 7 \bmod 10$. Many daily operations – counting in weeks, in months, in hours, … are very practical applications of modular arithmetic. But modular arithmetic is used extensively in computer sciences, in number theory (solutions of diophantine equations) in chemistry and in a multitude of other sciences. Here is a problem in modulo 10 arithmetic that will help you understand a magic trick. Mathusalem has a deck of 52 cards. He just performed a magic card trick for Mathilda. He dealt 51 cards face up, so that he could keep track of them and dealt the last one, the 52^{nd}, face down. The object of the game is to guess the value of the last card that was dealt. Each ten, jack, queen, and king is worth 10 points. Every ace is worth 1 point, every 2 is worth 2 points, … every 9 is worth 9 points. In his last card trick Mathusalem noticed that, after he had dealt 44 cards, the total number of points dealt was a multiple of 10 and that the next seven cards that were dealt were 3, 6, ace, queen, king, 4, and 8. What was the value of the 52^{nd} card that was dealt?

A) 1 B) 4 C) 10 D) 8 E) 5

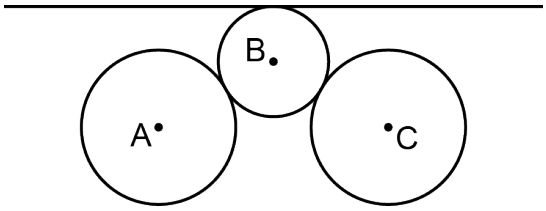
27. Each Wednesday, over a 49 day period, Mathusalem went to the opera. What is the maximum number of operas that he could have seen?

A) 6 B) 9 C) 5 D) 8 E) 7

28. What is the value of $8^{2/3}$?

A) 6 B) 20 1/3 C) 4 D) 5 1/3 E) 5

29. Points A, B, and C are the centres of 3 circles. Circle B (of radius r) is tangent to the straight line and to the two congruent circles A and C. The radius of these two circles is 1 and distance AC = 3. Line segment AC and the tangent to circle B are parallel. If the radius of circle B can vary (all other things being equal), what should the value of r be so that the tangent to circle B could also be tangent to circles A and C?



A) 1 B) 9/16 C) $\sqrt{3}/3$ D) 7/16 E) $\sqrt{7}/6$

30. Which of the following answers is closest to the perimeter of triangle ABC?

A) 15.9 u
B) 16.4 u
C) 16.1 u
D) 15.8 u
E) 16.5 u