Mathematica Centrum

Together, let's shape the mathematicians of the future

A. <u>You will need</u>:

1. A **blank response** form with the title "Mathematica", identical (except for the title) to the ones which are used for the Contests. Download this form and make as many copies as you need, so that your students can write the preparatory test and learn how to properly fill out a response form. (Remember that these copies cannot be used for the Contests. Your students will use the response forms that will be sent to you. Each student registered for a contest will receive a response form corresponding to the contest that he is writing. The only reason you are doing these copies is to show your students how to properly fill out a response form).

2. The **preparatory test** (this document), which your students can write to become familiar with multiple choice questions. Download this test and make as many copies as you need. (Remember that you are doing this to explain to your students the purpose of the preparatory test. The preparatory test defines the type of problems that appear in the actual contest.)

3. The **answer key**. Download the answer key and make as many copies as you need.

B. <u>How to fill out a response form properly</u>:

Use an **<u>HB pencil</u>** for coding all parts of the form. Do not use a ball point pen or felt-tip marker.

In the box at the top part of the form, tell your students to **PRINT** their school's name in full as well as their city/town and province. To the right of the box, tell them to **PRINT** their date of birth and sign their name to certify that the answers given represent their own work.

In the box on the mid-left of the form, tell your students to **PRINT** their last and first names. Tell them to code each letter by filling in the appropriate circle under each letter. (If your last name is Mathews, first you code the letter M by filling in the circle containing the M right under the letter M of Mathews, then you code the A by filling in the circle containing the A right under the letter A of Mathews. Do this for every other letter of your last name and for each letter of your first name). If the last name of a student is hyphenated, for example Jones-Smith, or if his/her first name is hyphenated, like Carol-Ann, inform the student to simply write Jones Smith and Carol Ann.

The mid-right part of the form outlines important instructions which are a reminder of what to do to code the response form correctly. The lower part of this box shows examples of incorrect coding. Remind your students to **completely** fill in each circle.

The box at the bottom of the form is made of circles which your students will fill in to record their answers to the questions. Again, tell them to fill each circle completely!

C. <u>Problems</u>:

Allow your students to write the preparatory test to be sure that they understand how to properly fill out the response form and to prepare them as to the type of problems that appear in the actual contests. It is important that your students do the problems intended for them :

Newton : all of the problems (except #21 and #30)
Lagrange : problems #1 to #25 (except #20)
Euler : problems #1 to #19 plus #24, #26, and #30

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PREPARATORY TEST 2010

EULER (7th) – LAGRANGE (8th) – NEWTON (9th)

1. The value of -5 + (-7) - (3 - 5) is

A) -10 B) 10 C) -14 D) 8 E) -12

2. What is the value of $2 + 2^3 + \sqrt{16}$?

A) 10 B) 12 C) 14 D) 16 E) 18

3. (5/8 + 1/2) ÷ (7/16 - 2/8) = ?

A) 1/2 B) 8 C) 4 D) 6 E) 27/108

4. The largest prime factor of 310 is

A) 2 B) 31 C) 5 D) 7 E) 11

5. If x% of 25 is 30, then x% of 20 is

A) x B) 12 C) 0.2 D) 18 E) 24

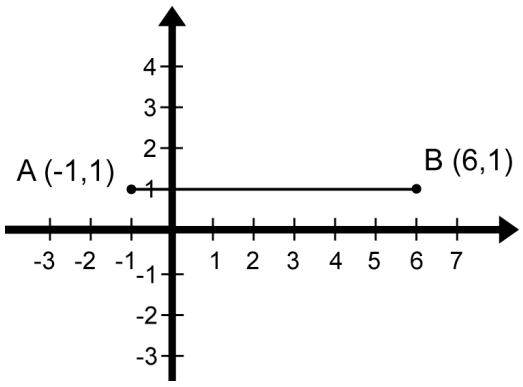
6. The product of two natural numbers is 12. Their largest possible sum is

A) 10 B) 11 C) 12 D) 14 E) 13

7. 1 L of liquid A has 10% more calories than 1 L of liquid B. How many calories does 1 L of liquid B contain, if 1 L of liquid A contains 660 calories?

A) 600 calories B) 550 calories
C) 726 calories D) 400 calories
E) 500 calories

8. Line segment AB is reflected in the x-axis. The coordinates of the images of points A and B after the reflection are, respectively,



A) (-1,-1) et (6,1) B) (-1,-1) et (6,-1)
C) (-1,1) et (1,6) D) (1,-1) et (-6,1)
E) (-1,1) et (6,-1)

9. For how many values of the set {-4, -2, -1/2, 0, 1/4, 2/3} is $x > x^2$?

A) 6 B) 3 C) 2 D) 4 E) 5

10. The ages of 5 people vary from 49 to 90 years old. Which of the following choices could represent their average age?

A) 84 B) 54 C) 57 D) 79 E) 82

11. Mathew has written all the natural numbers from 1 to 100. Which digit has he used exactly 11 times?

A) 5 B) 6 C) 4 D) 1 E) 0

12. The average of 5/6 and 8/12 plus the average of 1/3 and 1/2 is equal to

A) 4/3 B) 7/6 C) 7/12 D) 1 E) 3/2

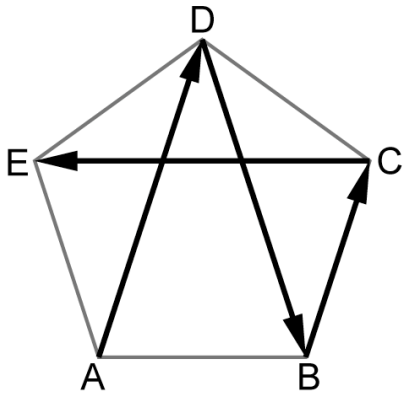
13. What is the smallest positive integer that is divisible by 3, 4, 6, and 8?

A) 30 B) 48 C) 24 D) 12 E) 36

14. A ball loses two thirds of its energy every time it hits a surface. After each bounce, it rises to one third the height from which it fell. After how many bounces will it rise to a height of less than 2 metres, if the ball initially falls from a height of 144 metres?

A) 5 B) 4 C) 3 D) 6 E) 2

15. Five towns forming the vertices of a pentagon are joined by a highway system as shown in the diagram. The highway system is made of diagonal highways (like AD) and lateral highways (like BC). Starting from A, a salesman wants to go to all other towns (B, C, and D) only once before he goes to E. The salesman must always travel in a straight line when going from one town to another. The diagram shows one of the paths (in bold) that the salesman can follow to go from A to E (ADBCE). In total, how many different ways can the salesman drive from A to E?



A) 4 B) 5 C) 6 D) 7 E) 10

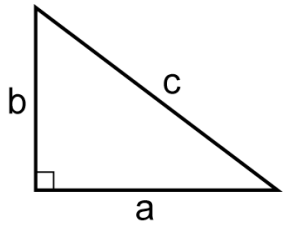
16. The result of (4 - 3) + (5 - 4) + (6 - 5) + … + (103 - 102) is

A) 100 B) 101 C) 99 D) 98 E) 102

17. How many 0's are used when $10^{20} + 999$ is written as a natural number?

A) 17 B) 18 C) 16 D) 19 E) 20

18. Pythagoras' theorem states that, in a right triangle, if c is the length of the hypotenuse and a and b are the lengths of the other two sides, then $a^2 + b^2 = c^2$. If a = 6 and b = 8, then $c^2 = 6^2 + 8^2 = 100$ and $c = \sqrt{100} = 10$. What is the length of c if b = 1 and a = 1?

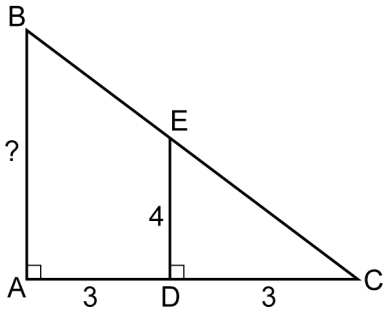


A) 2 B) 1.5 C) $\sqrt{3}$
D) $\sqrt{2}$ E) 0.41

19. The height of a rectangle is 3 cm and its base is 5 cm. What is the area of the new rectangle when the height is doubled and the base is tripled?

A) 15 cm^2 B) 65 cm^2 C) 60 cm^2
D) 90 cm^2 E) 42 cm

20. Two right triangles ABC and DEC are shown in the diagram. If AD = 3, DC = 3, and DE = 4, what is the length of line segment AB?



A) 6 B) 10
C) 5 D) 6.5
E) 8

21. A bag contains x red balls and y green balls. If a ball is chosen at random, what is the probability that the ball is green?

A) x : (x + y) B) y : (x + y) C) xy : (x + y) D) (x + y) : y E) (x + y) : x

22. If 3x + 2 = 2x - 2, what is the value of 2x + 5?

A) 3 B) 1 C) -2 D) -1 E) -3

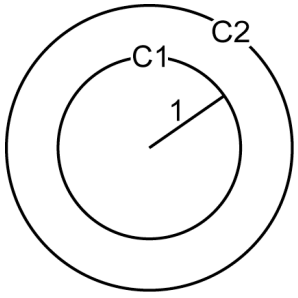
23. Two positive integers are in the ratio 5 : 3. If their difference is 12, what is their sum?

A) 18 B) 30 C) 48 D) 40 E) 50

24. When the digits 1, 2, 3, and 4 are all used to form 4-digit natural numbers, 24 different numbers can be formed. If all these numbers are written in decreasing order (from the largest to the smallest), what number will be the 6th number on the list?

A) 3 241 B) 3 421 C) 4 132 D) 4 123 E) 4 312

25. Two circles (C1 and C2) are concentric. The area of the large circle is triple the area of the smaller one. What is the difference between the radii of the two circles, if the radius of the smaller circle is 1?



A) 2 B) $\sqrt{2}$ C) $\sqrt{3}$
D) $\sqrt{3} - 1$ E) $\sqrt{2} - 1$

26. Let m be a natural number. If we know that m^2 has 10 prime factors, how many prime factors does the number m have?

A) 5 B) 4 C) 10 D) 6 E) 20

27. There are two numbers on the number line that are twice as far from 2 as from -1. If the number 0 is one of them, what is the other?

A) 1/2 B) -3/4 C) -4 D) -3 E) 6

28. What is the product of (x + 2) (x + 2)?

A) $x^2 + 4x + 4$ B) $x^2 + 2$ C) $x^2 + x + 4$ D) $x^2 + 2x + 4$ E) $x^2 + 4$

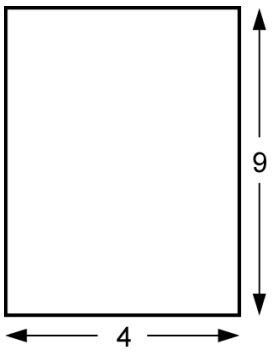
29. The base of an isosceles triangle is 6 cm and its area is 12 cm^2. What is its perimeter?

A) 18 cm B) 15 cm C) 13 cm D) 12 cm E) 16 cm

30. Did you know that we can find the number of divisors of any natural number without knowing each one of its divisors? How? First, the number must be factored. Every natural number n can be factored as $n = p_1^{E1} \times p_2^{E2} \times p_3^{E3} \ldots \times p_n^{En}$. In this factorisation, the symbols p_1, p_2, p_3, …, p_n represent the different prime factors of the factorisation, while E1, E2, E3, …, En represent the respective exponents of these prime factors. The factorisation of 24 is $2^3 \times 3^1$, that of 36 is $2^2 \times 3^2$. The exponents of the prime factors of a number enable us to find the number of its divisors by simply replacing the values of E1, E2, …, En corresponding to that number in the expression: (E1 + 1) x (E2 + 1) x (E3 + 1) … x (En + 1). The number of divisors of 24 is given by the product (3 + 1) x (1 + 1), which is 8. By the way, the 8 divisors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. The number of divisors of 36 is given by the product (2 + 1) x (2 + 1), which is 9. The 9 divisors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. We can find the number of divisors of any natural number without knowing each one of its divisors. What is the number of divisors of 300?

A) 9 B) 18 C) 12
D) 14 E) 4

31. The volume of a right cylinder is given by the formula $V = \pi r^2 h$ and that of a sphere by the formula $V = 4/3\ \pi r^3$. What is the radius of the sphere that has the same volume as the right cylinder whose front view is shown in the diagram?



A) π B) 2 C) 3
D) 3π E) √π

32. If x/3 = y/4, what is the value of (8x + 6y) : 2y?

A) 4 B) 6 C) 5 D) 3 E) 7