Mathematica Centrum

Together, let's shape the mathematicians of the future

PREPARATORY TEST 2011
COMPLETE SOLUTIONS

THALES (3rd) – BYRON-GERMAIN (4th) – FIBONACCI(5th) – PYTHAGORAS (6th)

1. The number of edges of a cylinder (2) plus the number of sides of a rectangle (4) is equal to 6.

2. The value of X in the equation: X + 2 = 10 is 8.

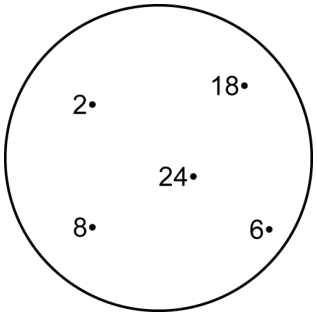
3. The value of the unknown number in the equation: 12 x ? = 14 + 10 (12 x 2 = 24) is 2.

4. An ''average'' person sleeps about 8 hours per day. In one month a person sleeps approximately (30 x 8) 240 hours.

5. 400 nickels = 200 dimes (400 x 5 = 200 x 10).

6. 6 ÷ 2 + 5 x 2 = 3 + 5 x 2 = 3 + 10 = 13.

7. The elements 2, 6, 8, and 24 are all divisors of 24. The number 24 is not divisible by 18.

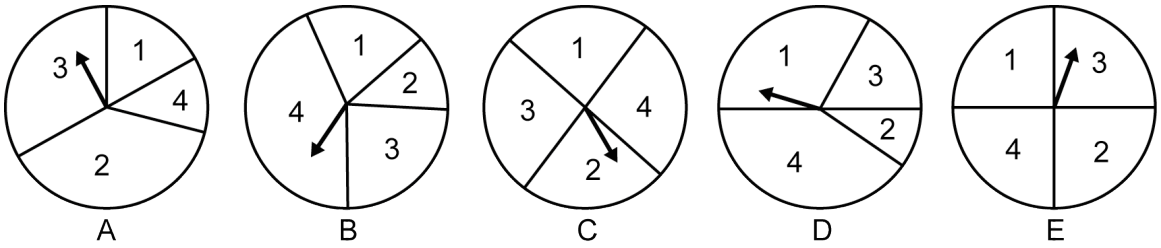


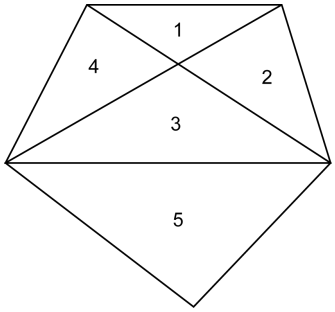
8. Because Mathew is 2 years older than Melissa and 4 years older than Mathilda, we can conclude that Melissa is 2 years older than Mathilda. Melissa is 2 years older than Mathilda and 2 years younger than Mathew. If Mathilda were 2 years older and Mathew 2 years younger, all 3 would have the same age. The sum of their ages would still be 18 years (18 + 2 - 2) and each one would be 6 years old. Melissa is 6, Mathew is 8, and Mathilda is 4 years old.

9. 10 hundreds (10 x 100 = 1 000) - 70 ones (70) + 27 tens (27 x 10 = 270) is equal to (1 000 - 70 + 270) 1200.

10. The largest natural number that, multiplied by 7 (20 x 7 = 140), gives a result smaller than 143 is 20 (21 x 7 = 147).

11. The spinners E and C give the same chances of getting a 3 or a 2 as of getting a 1 or a 4 (the area of sectors 3 and 2 is the same as that of sectors 1 and 4, which is the area of a half-circle). The spinners D and B increase the chances of getting a 1 or a 4 whereas spinner A gives more chances of getting a 3 or a 2 than that of getting a 1 or a 4 (the area of sectors 3 and 2 is more than a half-circle).





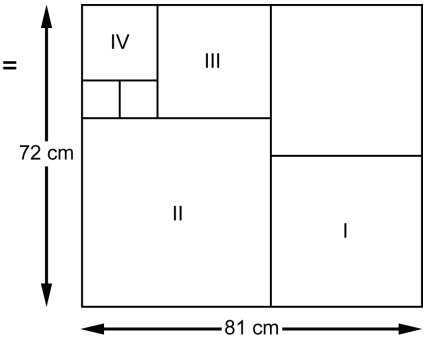
12. There are 9 triangles in the diagram: the 5 smaller triangles 1, 2, 3, 4, and 5 plus all the other triangles formed by a combination of two of these smaller triangles (triangles 1-2, 2-3, 1-4, and 3-4).

13. The value of (1 + 2 + 3 + … + 20) is 210. The value of (2 + 4 + 6 + … + 40) is double that of the first series because each term in the second series is double that of the corresponding term in the first series. The value of the second series is (2 x 210) 420.

14. If 3 x 37 = 111, then 27 x 37 is equal to (9 x 3) x 37 = 9 x (3 x 37) = 9 x 111 = 999.

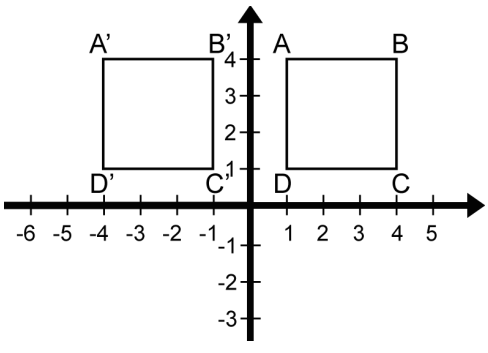
15. The sum of 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 is equal to 36 and 36 is not divisible by 7.

16. The length of the side of square tile I is (72 ÷ 2) 36 cm, that of tile II is (81 cm - 36 cm) 45 cm. The side of tile III is (72 cm - 45 cm) 27 cm. The side of tile IV is (45 cm - 27 cm) 18 cm. The length of of the side of the smallest tile is (18 cm ÷ 2) 9 cm.



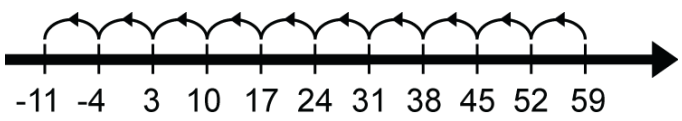
17. The greatest factor of 12 (1, 2, 3, 4, 6, and 12) is 12. Because 36 and 72 are also divisible by 12 (36 ÷ 12 = 3 and 72 ÷ 12 = 6), the GCF of 12, 36, and 72 is necessarily 12.

18. Square A'B'C'D' is the image of square ABCD when it is slid 5 units to the left. The coordinates of point D' are (-4, 1).

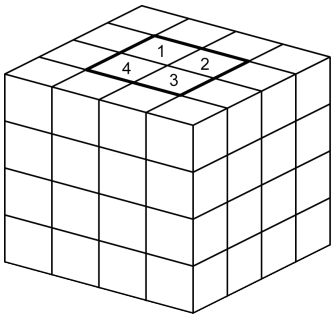


19. There are 3 prime numbers between 57 and 69 (59, 61, and 67).

20. Starting at 59 on the number line, you must subtract '7' 10 times (10 x 70 = 70) because 59 - 70 is equal to -11 (see diagram below).



21. The pile shown is made of 64 cubes. The faces enclosed in the bold type rectangle are those of the 4 cubes of which only one face can be seen. (there are 4 more at the bottom of the large cube). Between these two layers of 4 cubes of which only one face can be seen, there are (2 x 4) 8 cubes whose faces are completely hidden.



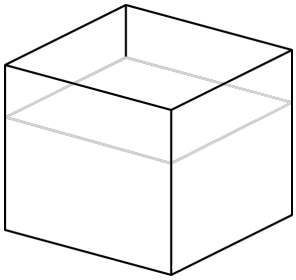
22. The only even prime number is 2. Therefore, there is only one prime number between 1 and 1 000 that has a one's digit which is a 2.

23. The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. The second largest factor of 36 is 18.

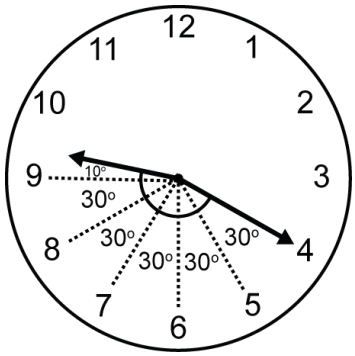
24. If N/M = 0.3, then 7N/M = 7 x 0.3 = 2.1 and 7N/3M = 2.1 ÷ 3 = 0.7.

25. The one's digit of the sum of all the natural numbers from 0 to 11 (0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 0 + 1 = 46) is 6.

26. A cubic tank 20 cm x 20 cm x 20 cm has a volume of (20 cm x 20 cm x 20 cm) 8 000 cm^3. When it is 3/4 full (3/4 of 8 000 cm^3), its volume is 6 000 cm^3. When it is 7/8 full (7/8 of 8 000 cm^3), its volume is 7 000 cm^3. 1 000 cm^3 (7 000 cm^3 - 6 000 cm^3) of water has to be poured into the tank so that it is 7/8 full.



27. When the minute hand goes around 360°, the hour hand moves (360° ÷ 12) 30°. Given that it is 9:20, the minute hand has gone one third of the way around while the hour hand has turned 1/3 of 30°, or 10°. At 9:20, the minute and hour hands form a (5 x 30° + 10°) 160° angle.



28. Of the five months that are given, only January has 31 days (April, June, and September have 30 days, February has 28 or 29) .

29. Mathew ran the first 60 metres in (60 m ÷ 10 m/s) 6 seconds and the last 140 metres in (140 m ÷ 7 m/s) 20 seconds. Mathew finished the race in (6 s + 20 s) 26 s.

30. G must be greater than 4, but it must also be even because it is the result of the addition of two C's. G could be 6 or 8, but it cannot be equal to 8 because C then would be equal to 4 or 9. C cannot be equal to 9 because the problem states that it is even. It cannot be equal to 4 because F would be even (with no carry over F would be the result of B + B). G must be equal to 6 and C must be equal to 8. E must be equal to 1 because the sum of the numbers ABC and DBC cannot be greater than 1 998 (999 + 999 = 1 998). F is odd and greater than 5. It could be either 7 or 9. It cannot be 9 because B then would be equal to 4 and the number EBFG would be 1 496. To get this number, we would need to have A = 9 and D = 4 (A > D) and consequently the values of A and F would both be equal to 9. F must be equal to 7 and B must be equal to 3. The number ABC is 938, the number DBC is 438 and EBFG is 1 376.

$$\begin{array}{rr} & A\ B\ C \\ + & D\ B\ C \\ \hline & E\ B\ F\ G \end{array}$$

31. There ages will never be 3 prime numbers again. The year later, their ages will be 3, 6, and 8 years old. Two years later, their ages will be 4, 7, and 9. When the age of the youngest is even, the age of the two others is odd (2, 5, and 7 or 4, 7, and 9). When the age of the youngest is odd, the age of the two others is even (3, 6, and 8). Every birthday, there will always be one of the three brothers whose age is even. Because 2 is the only even prime number, their ages will never be represented again by 3 prime numbers.