Mathematica Centrum

Together, let's shape the mathematicians of the future

PREPARATORY TEST 2011
COMPLETE SOLUTIONS

EULER (7th) – LAGRANGE (8th) – NEWTON (9th)

1. The sum of the prime factors of 20 (2 x 2 x 5) is 9.

2. The value of Y is 150°, that of X is (180° - 150°) 30°. The value of 2Y + X is 330°.

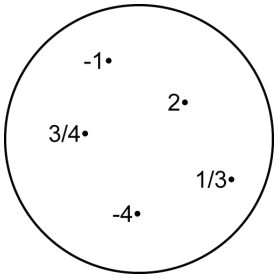
3. April 14 2011 (2011 is not a leap year) will come 365 days (52 x 7 + 1) after April 14 2010. This day will come 52 weeks and 1 day after April 14 2010. It will fall on a Thursday.

4. The result of 0.1 x 0.1 x 0.1 x 10 is 0.01, which is 1%.

5. The exception is 12 (1, 2, 3, 4, 6, 12) which has 6 factors.

6. A number divided by 3 leaves a remainder of 1. This number is of the form 3N + 1. Replacing N by 0, 1, 2, 3, … we get the following set of number {**1**, 4, 7, 10, 13, 16, 19, 22, …}. Another number divided by 3 leaves a remainder of 2. This number is of the form 3N + 2. Replacing N by 0, 1, 2, 3, … we get the following set of numbers {2, **5**, 8, 11, 14,17, 20, 23, …}. The product of these 2 numbers could not be 25 because this number can only be obtained by the product of 5 x 5 or 1 x 25.

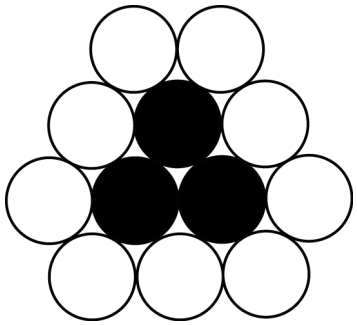
7. 5 1/3 x 2 1/4 x 3/8 x 4/9 = 16/3 x 9/4 x 3/8 x 4/9 = 16/8 = 2.



7. No element of the set verifies $x^2 < -x$. Actually $(-1)^2 = 1$, $2^2 > -2$, $(3/4)^2 > -3/4$, $(-4)^2 > 4$ and $(1/3)^2 > -1/3$.

9. It takes 9 circles to completely surround the 3 tangent circles.

10. The 4-digit perfect square 2mn1 could be (51^2) 2**60**1 or (49^2) 2**40**1. The possible value of m + n is 6 or 4.



11. The sum of 2 integers is 1 and their product Is -2. The factors of 2 are 1 and 2. We can get a product of -2 in two different ways: 1 x -2 and -1 x 2. The sum of the two numbers being 1, we can conclude that the two numbers are 2 and -1. Their quotient could be -2 or -1/2.

12. The number of seconds in one hour is (1h = 60min = 60 x 60s) 3 600 seconds. 3 600 minutes are equal to 60 hours (60h = 60 x 60min = 3 600min).

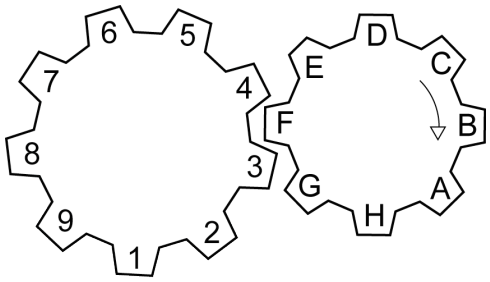
13. The values of y lie between 0 and 3. Therefore y is always positive. The values of x lie between -4 and 0. Therefore x is always negative. The answer y - x >0 (y > x) is always true.

14. The average of -1/2 and 1/3 (-1/2 + 1/3 = -3/6 + 2/6 = -1/6 and -1/6 ÷ 2 = -1/12) is -1/12.

15. The distance that the car covers when each tire makes 100 revolutions is (100 x π x 1) 100π, which is approximately 314 m.

16. There are 5 balls left in the triangle. Mathilda cannot win the game. Actually, if she removes 1 ball, Mathew removes 3 balls (taking the 14^{th} ball) and wins the game. If Mathilda takes out 2 balls, Mathew takes out 2 balls and wins again. If Mathilda removes 3 balls from the triangle, Mathew wins again by removing 1 ball. Mathew was certain to win because he removed the 10^{th} ball from the triangle. The person who removes the 10^{th} ball is certain to remove the 14^{th} ball. The person who removes the 6^{th} ball is certain to remove the 10^{th}. Finally, the person who takes out the 2^{nd} ball is sure to remove the 6^{th} one. Applying this method, the player who plays first is in a position to win any game.

17. The result of $1 + 3 = 4 = 2^2$. The result of $1 + 3 + 5 = 9 = 3^2$. The result of $1 + 3 + 5 + 7 = 4^2 = 16$. The sum of the series of the first ''n'' consecutive odd numbers is always n^2. To know how many terms there are in a series of consecutive odd numbers just calculate the half sum of the first and last terms of the series. The series 1 + 3 + 5 + 7 … + 41 has ((1 + 41) ÷ 2) 21 terms. The result of this series is 21^2 (441).



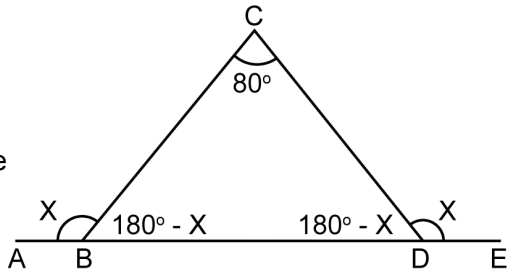
18. Tooth F is meshed with teeth 3 and 4. When the gear on the right turns in the direction of the arrow, the lettered teeth turn in the following ordered sequence: F, G, H, A, B, C….The numbered teeth of the left gear turn in the following ordered sequence: 3-4, 2-3, 1-2, 9-1, 8-9, …. The sequence of the meshed teeth will be 3F4, 2G3, 1H2, 9A1 … . The teeth that will be meshed when tooth A falls into the position now occupied by tooth F are 1A9.

19. Four neighbours have bought a snow blower and have shared the cost equally. If x is the price of the snow blower, each neighbour has paid x/4. If there were only three neighbours, each one would have paid x/3. We can write that x/3 - x/4 = 105, 4x/12 - 3x/12 = 105, 4x - 3x = 1 260 and x = $1 260. If 5 neighbours had bought the blower, each one would have paid (1 260 ÷ 5) $252.

20. The radius of the circle is 2. The length of the circle is (2π2) 4π. The length of a quarter-circle is π.

21. It's average speed is (150 km ÷ 15min) 10 km/min or 600 km/h.

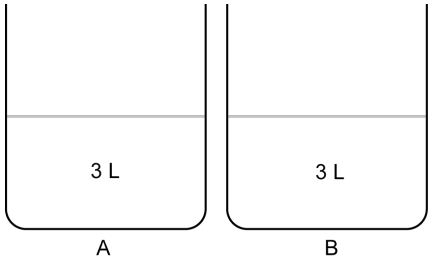
22. Mathew has received 1 + 3 + 5 + … = 1 000 000. We know that n^2 = 1 000 000 and that n = 1 000. He has received a certain amount (1, 3, 5, …) every week for 1 000 weeks. The amount x received on the $1\,000^{th}$ week is given by the equation: (1 + x) ÷ 2 = 1 000 (see number 17). This equation becomes 1 + x = 2 000. We find that x = 1 999. The last week, Mathew received $1 999.



23. The value of X is given by equation 180° - X + 180° - X + 80° = 180°. We find that X = 130°.

24. The value of 2^{25} is $2^5 \times 2^5 \times 2^5 \times 2^5 \times 2^5 = 32 \times 32 \times 32 \times 32 \times 32 = 33\,554\,432 = 3.4 \times 10^7$. The closest answer is 10^7.

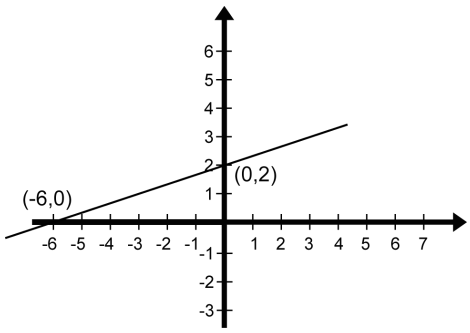
25. Pythagoras' theorem states that, in a right triangle, if c is the length of the hypotenuse and a and b are the the lengths of the other two sides, then $a^2 + b^2 = c^2$. If a and b are two sides of a square (a = b = 1) and c is the length of a diagonal, then $c^2 = 1^2 + 1^2 = 2$ and $c = \sqrt{2}$.



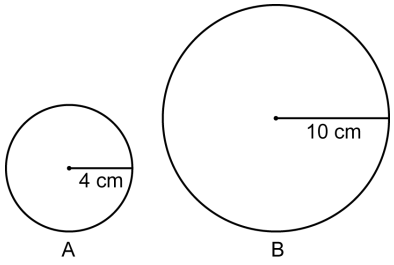
26. Container A holds 3 litres of water. Recipient B holds a homogeneous mixture of 2 litres of wine and 1 litre of water. A litre of mixture B is poured in container A. This litre contains 2/3 of a litre of wine and 1/3 of a litre of water. Recipient A now contains 3 1/3 litres of water and 2/3 of a litre of wine. Recipient A contains a total of (3 1/3 + 2/3) 4 litres of a mixture of water and wine. Wine represents (2/3 ÷ 4 = 2/12) 1/6 of the mixture in A.

27. Because I get a remainder of 1 each time I divide the number by 2, 3, 4, or 5, this number must be of the form 60N + 1. If the number was divisible by 2, 3, 4, and 5 (2 x 3 x 2 x 5), it would be a multiple of 60 (it would be of the form 60N). However, there is a remainder of 1, so it must be of the form 60N + 1. Replacing N by 0, 1, 2, 3, 4, … we get the numbers 1, 61, 121, 181, 241, … . The number we are looking for is 181 and the sum of its digits is (1 + 8 + 1) 10.

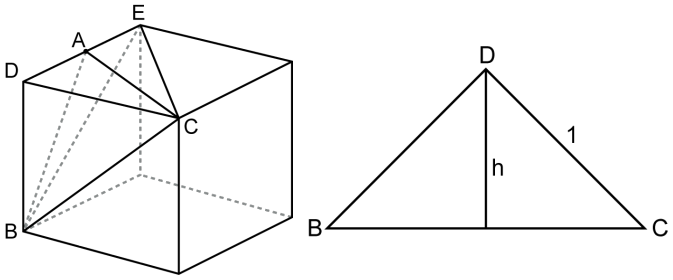
28. The graph of line 3y - x = 6 is shown in the diagram. The number of grid points on this line is given by the equation y = x/3 + 2. This equation tells us that y is an integer only when x is a multiple of 3. For the interval $-6 \le x \le 4$, y is an integer for x = {-6, -3, 0, 3}. There are 4 grid points on this line for $-6 \le x \le 4$.



29. The volume of cylinder A is $(\pi 4^2 \times 20)$ $320\ \pi\ cm^3$. The volume of B is given by the equation $\pi 10^2 \times h = 320\pi$. This equation becomes 100h = 320, which gives h= 3.2 cm.



30. The edges of the cube shown in the diagram have a value of 1. Points B, C, D, and E are 4 vertices of the cube, whereas point A is a point of one of its edges. Triangle BDC is an isosceles right triangle (angle D = 90° and DA = DC = 1). The length of line segment BC is given by $BC^2 = 1^2 + 1^2$. The value of segment BC is $\sqrt{2}$. Now look at triangle BDC in which we have drawn the height h of side BC. Again, we can write that $h^2 + (\sqrt{2}/2)^2 = 1^2$. We find that $h = \sqrt{2}/2$. By the way, triangles ABC and EBC are also isosceles triangles. Triangle EBC is in reality an equilateral triangle because its sides are all equal to $\sqrt{2}$.



31. Given that all the pizzas have the same thickness and are covered by the same toppings, the best buy is given by the cost per cm^2 of pizza. Member D has made the best buy because he paid his pizza a mere 2.31¢ per cm^2 (E made the second best buy because he paid only 2.38¢/cm^2.)