Mathematica Centrum

Together, let's shape the mathematicians of the future

PREPARATORY TEST 2010
COMPLETE SOLUTIONS

THALES (3rd) – BYRON-GERMAIN (4th) – FIBONACCI(5th) – PYTHAGORAS (6th)

1. The number of vertices of a triangular pyramid is 4 (the 3 vertices of the base triangle and the vertex of the pyramid itself).

2. The value of X in the equation: X + 13 = 27 is 24.

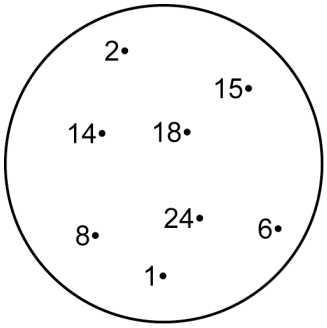
3. Eighteen plus twenty-seven is equal to 45.

4. The third letter before the 14th letter of the alphabet is the 11th letter of the alphabet. The vowel closest to this letter is the I.

5. 2 x 2 x 10 x 5 x 5 = 2 x 5 x 2 x 5 x 10 = 10 x 10 x 10 = 1 000.

6. Half of 24 is 12. The double of 3 is 6. The result of 12 ÷ 6 is 2.

7. Each quarter is equal in value to 5 nickels. 20 quarters are equivalent to (20 x 5) 100 nickels.

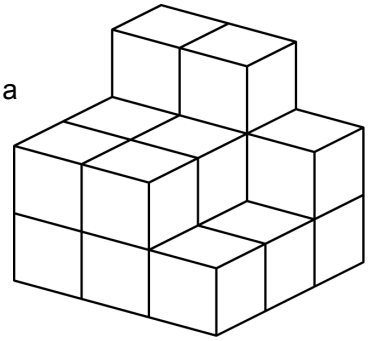


8. 8 - 4 ÷ 2 + 4 = 8 - (4 ÷ 2) + 4 = 8 - 2 + 4 = 10.

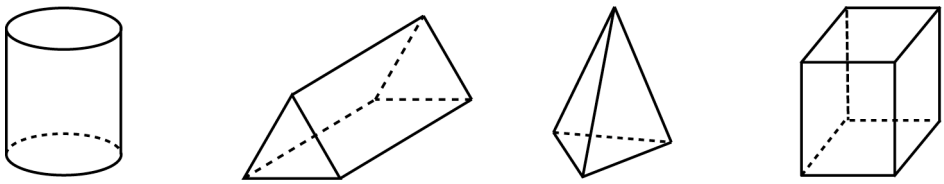
9. The numbers 1, 2, 6, and 18 are divisors of 36. The number of elements of the set that are not divisors of 36 is (8 - 4) 4.

10. The next number in the sequence: 10, 20, 18, 36, 34, … is 68 (10 x **2** = 20, 20 - **2** = 18, 18 x **2** = 36 …). The rule of the sequence is **x 2 - 2**).

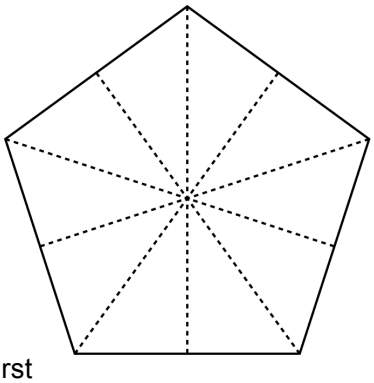
11. Nine blocks have to be added to the pile of blocks to get a cubic pile of 27 blocs (3 x 3 x 3). Ten blocks have to be subtracted from the pile to get a cubic pile of 8 blocs (2 x 2 x 2). The minimum number of blocks that have to be added to, or subtracted from, the pile of 18 identical blocks shown on the right to get a pile of blocks that will have the shape of a cube is 9.



12. The first solid is a cylinder, the second is a triangular prism (the two parallel bases are triangles), the third is a triangular pyramid, and the fourth is a rectangular prism. The number of solids that are not prisms is 2.



13. The regular pentagon shown has 5 lines of symmetry.

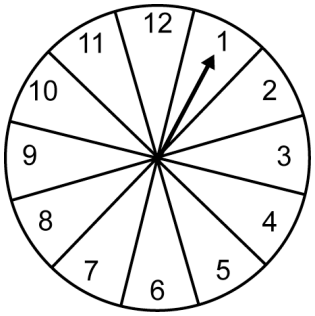


14. The numbers 1, 4, 9, and 16 are square numbers (because 1 x 1 = 1, 2 x 2 = 4, 3 x 3 = 9, and 4 x 4 = 16). The square numbers between 0 and 100 are 1, 4, 9, 16, 25, 36, 49, 64, and 81. There are 9 square numbers between 0 and 100.

15. In 2 hours there are (2 x 60) 120 minutes. In 120 minutes there are (120 x 60) 7 200 seconds.

16. The largest of these numbers is 4 321. The 2nd largest is 4 312. In all, there are 6 numbers that start with the digit 4 (4 321, 4 312, 4 231, 4 213, 4 132, and 4 123). There are 6 other numbers that start with the digit 3, six others whose first digit is a 2, and finally, 6 others whose first digit is a 1. In all, 24 different numbers can be formed.

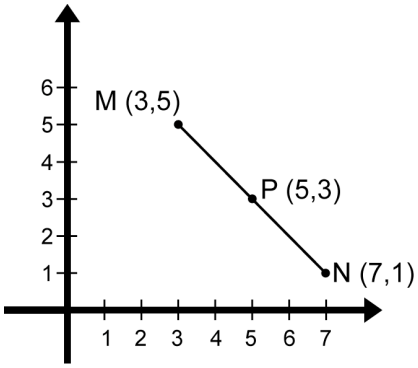
17. The first even number is 0 (1 x 2 - 2), the second is 2 (2 x 2 - 2), the third is 4 (3 x 2 - 2), the fourth is 6 (4 x 2 - 2), the fifth is 8 (5 x 2 - 2) , … the 15th even number is (15 x 2 - 2) 28.



18. Andrea got home at 1:41 P.M. She read for 30 minutes. Then she ate for 45 minutes and finally, she studied for 1 h 15 min. She finished studying (30 + 45 + 75) 150 minutes later. She finished studying at (13 h 41 min + 2 h 30 min) 4:11 P.M.

19. There are 5 prime numbers on the spinner (2, 3, 5, 7, and 11). If she spins the spinner only once, she will have 5 chances to get a prime number. The probability that she will get a prime number is 5/12.

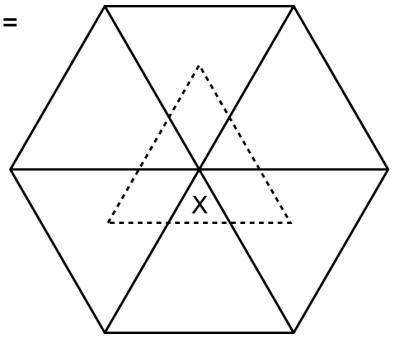
20. The mid point P on segment MN is represented in the diagram. Its coordinates are (5 , 3). The x coordinate of this point must be midway between 3 and 7 (3 + 7 = 10, 10 ÷ 2 = 5). The y coordinate of this point must be midway between 1 and 5 (1 + 5 = 6, 6 ÷ 2 = 3).



21. The prime factors of 12 are (2 x 2 x 3). The prime factors of 15 is (3 x 5). The LCM of 12 and 15 are (2 x 2 x 3 x 5) 60.

22. The fractions that are equivalent to 1/5 (2/10, 101/505, and 14/70) are all those whose denominator is 5 times greater than the numerator (10 = 5 x 2, 505 = 5 x 101, …). Three fractions are equivalent to 1/5.

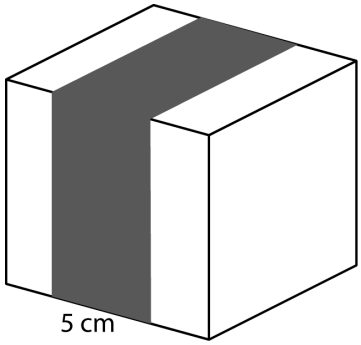
23. The equation 2 x N - 4 x 1.3 = 15.2 becomes 2 x N - 5,2 = 15.2 then 2 x N = 20.4, and finally N = 10.2. The value of N in this equation is therefore 10.2.



24. The only natural number between 0 and 40 that is a multiple of 2, 3, and 5 is (2 x 3 x 5) 30.

25. The maximum number of triangles that she can place on triangle X (to cover it completely) is (360 ÷ 6) 6, because each angle of an equilateral triangle is equal to 60° and each circle has 360°. The six triangles are placed as shown in the diagram.

26. The area of one of the cube's faces is (10 cm x 10 cm) 100 cm^2. The cube's total surface is 600 cm^2. On each face, Mathew applied (5 cm x 10 cm) 50 cm^2 of paint. In all, he painted a surface of (4 x 50) 200 cm^2. He painted (200 cm^2 ÷ 600 cm^2) 1/3 of the total surface of the cube.

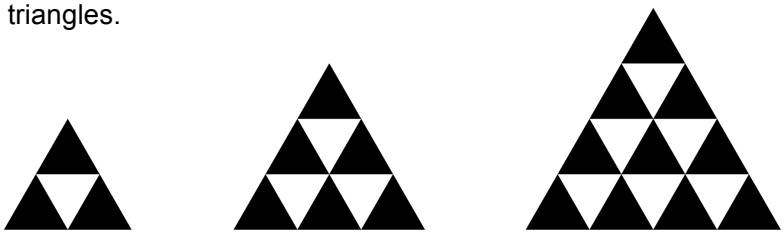


27. The numbers 6, 120, 24, and 60 can all be written as the products of 3 natural consecutive numbers (6 = 1 x 2 x 3, 120 = 4 x 5 x 6, 24 = 2 x 3 x 4, and 60 = 3 x 4 x 5). It is impossible to write the number 12 as the product of 3 consecutive natural numbers.

28. The product of 9 x 10 x 11 x 101 can be written as 9 x 11 x 101 x 10 = 99 x 101 x 10 = 9 999 x 10. This number is smaller than 10 000 x 10 (which is 10^5) and larger than 1 000 x 10 (which is 10^4). This product lies between 10^4 and 10^5.

29. All the numbers are the products of 2 prime numbers, except 20, which is the product of 3 prime numbers (2 x 2 x 5).

30. The first 3 figures in a sequence are represented in the diagram. The first figure is composed of **3** shaded triangles, the second figure is composed of **6** shaded triangles (**3** + 3), and the third is composed of **10** triangles (**6** + 4). The 4th is composed of (**10** + 5) **15**, the 5th of (**15** + 6) **21** shaded triangles, … the 10th figure in the sequence has **66** shaded triangles.



3 4 5 6 7 8 9 10 11

3 6 10 15 21 28 36 45 55 66

31. In the first race, Mathilda finished 5 m ahead of Mathew. When Mathilda finished the race, Mathew had covered (95/100) 95% of Mathilda's distance. If they run a second race of 99 m at exactly the same speed they ran the first one, Mathida will again finish the race when Mathew will have covered 95% of Mathilda's distance. In the second race, when Mathilda crosses the finish line, Mathew will have covered (0.95 x 99) 94.05 m. Mathida will finish (99 – 94.05) 4.95 m ahead of Mathew.