Mathematica Centrum

Together, let's shape the mathematicians of the future

PREPARATORY TEST 2010
COMPLETE SOLUTIONS

EULER (7th) – LAGRANGE (8th) – NEWTON (9th)

1. The value of -5 + (-7) - (3 - 5) is -5 - 7 - (-2) = -12 + 2 = -10.

2. The value of $2 + 2^3 + \sqrt{16}$ is 2 + 8 + 4 = 14.

3. (5/8 + 1/2) ÷ (7/16 - 2/8) = (5/8 + 4/8) ÷ (7/16 - 4/16) = 9/8 ÷ 3/16 = 9/8 x 16/3 = 6.

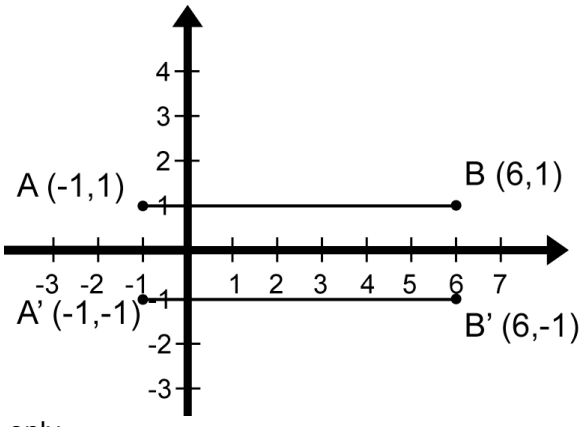
4. The prime factors of 310 are 2 x 5 x 31. The largest prime factor of 310 is 31.

5. If x% of 25 is 30, we can write that x% x 25 = 30, x% = 30/25, x% = 6/5, x% = 120%, and x =120. 120% of 20 = 1.2 x 20 = 24.

6. The product of two natural numbers is 12. These two numbers can be 1 and 12, 2 and 6, or 3 and 4. Their largest possible sum is (1 + 12) 13.

7. If 1 L of liquid A contains 10% more calories than 1 L of liquid B, then 1 L of liquid A contains 1.1 times more calories than 1 L of liquid B. If 1 L of liquid A contains 660 calories, then 1 L of liquid B contains (660 ÷ 1.1) 600 calories.

8. Line segment AB is reflected in the x-axis. The coordinates of the images of points A and B after the reflection are, respectively, (-1,-1) and (6,-1).



9. The only values for which $x > x^2$ are 1/4 and 2/3 (1/4 > 1/16 and 2/3 > 4/9). The values for which $x < x^2$ are -4, -2, and -1/2 (-4 < 16, -2 < 4, and -1/2 < 1/4). For x = 0, $0 = 0^2$.

10. The minimal average age of these 5 people is (4 x 49 + 90) ÷ 5 = 57.2 years. The maximum average is (4 x 90 + 49) ÷ 5 = 81.8 years. The only choice that could represent their average age is 79 years.

11. The only digit that he used exactly 11 times is the 0 (1**0**, 2**0**, 3**0**, … 1**00**).

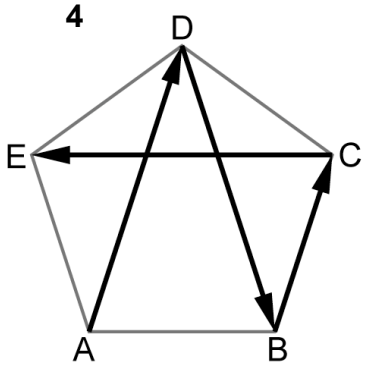
12. The average of 5/6 and 8/12 is ((10/12 + 8/12) ÷ 2) 9/12. The average of 1/3 and 1/2 is ((2/6 + 3/6) ÷2) 5/12. The sum of the two averages is (9/12 + 5/12) 14/12 or 7/6.

13. The smallest positive integer that is divisible by 3, 4, 6, and 8 is the LCM of these numbers. The LCM of 3 (3), 4 (2 x 2), 6 (2 x 3), and 8 (2 x 2 x 2) is (2 x 2 x 2 x 3) 24.

14. Look at the diagram below. After each bounce, the ball rises to one third the height from which it fell. After the first bounce, it rises to a height of 48 metres. After the second, it rises to a height of 16 metres, … . It will rise to a height of less than 2 metres after 4 bounces.

144 48 16 5.33 1.78
1 2 3 4

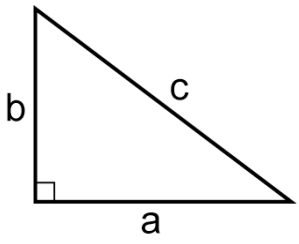
15. The diagram shows one of the paths (in bold) that the salesman can follow to go from A to E (A**DBC**E). In all, he can drive from A to E in 6 different ways (A**DBC**E, A**DCB**E, A**BCD**E, A**BDC**E, A**CDB**E et A**CBD**E). There are 6 different ways to go from A to E because there are 6 (1 x 2 x 3) different ways to arrange the letters **BCD**.



16. The result of (4 - 3) + (5 - 4) + (6 - 5) + … + (103 - 102) is 100. We know that each parenthesis is worth 1. We must find out how many parentheses there are. First, we can write this series as (4 - 3) + (5 - 4) + (6 - 5) + … + (100 - 99) + (101 - 100) + (102 - 101) + (103 - 102). The last 3 parentheses are equivalent to (1 - 0) + (2 - 1) + (3 - 2). If these 3 parentheses are placed at the beginning of the series, we get the following series: (1 - 0) + (2 - 1) + (3 - 2) + (4 - 3) + (5 - 4) + (6 - 5) + … + (100 - 99). This series is composed of exactly 100 terms. Each term being equal to 1, the result is 100.

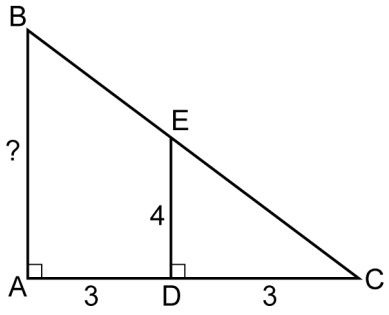
17. When 10^{20} + 999 is written as a natural number, we get the number (999 + 100 000 000 000 000 000 000) 100 000 000 000 000 000 999. This number has 17 zeros.

18. Pythagoras' theorem states that, in a right triangle, if c is the length of the hypotenuse and a and b represent the lengths of the other two sides, then $a^2 + b^2 = c^2$. If b = 1 and a = 1, then $c^2 = 1^2 + 1^2 = 2$ and $c = \sqrt{2}$.



19. The height of a rectangle is 3 cm and its base is 5 cm. When the height is doubled (6 cm) and the base is tripled (15 cm), the area of the new rectangle is (6 cm x 15 cm) 90 cm^2.

20. Triangles ABC and DEC are similar (point C is the centre of enlargement). The scale factor is 2, because AC : DC = 2. The length of line segment AB is (2 x ED = 2 x 4) 8.



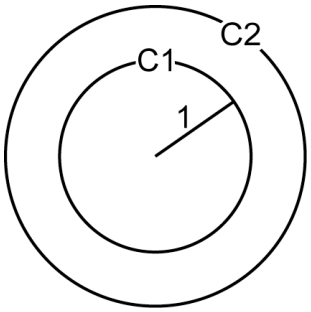
21. A bag contains x red balls and y green balls. If a ball is chosen at random, the probability that the ball is green is obviously y : (x + y).

22. From 3x + 2 = 2x - 2, we get x = -4. The value of 2x + 5 is 2 (-4) + 5 = -3.

23. Two positive integers are in the ratio 5 : 3. If the smaller one is x, then the larger one is x + 12. We can write the equation (x +12) : x = 5 : 3. This equation becomes 3(x + 12) = 5x, from which we get x = 18. The sum of these integers is (18 + 30) 48.

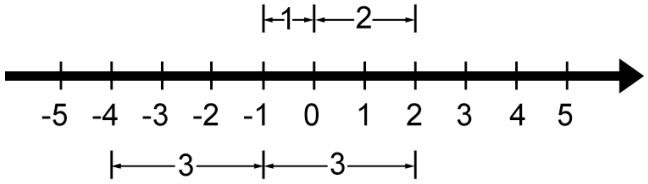
24. The largest of these numbers is 4 321. The 2nd is 4 312, the 3rd is 4 231, the 4th is 4 213, the 5^{th} is 4 132, and the 6th is 4 123.

25. The area of the smaller circle is $\pi\ 1^2 = \pi$. The area of the large circle is 3 x π. The radius of the large circle is given by the equation: $\pi\ r^2 = 3\pi$. We find $r = \sqrt{3}$. The difference between the two radii is $\sqrt{3} - 1$. By the way, this value represents the smallest possible distance between a point on C1 and a point on C2.



26. Let us suppose that m is 20. The number of prime factors of 20 is (2 x 2 x 5) 3. The number of prime factors of 400 is (2 x 2 x 2 x 2 x 5 x 5) 6. The number of prime factors of 20^2 is double that of 20. The number of prime factors of m^2 is always double that of m. If the number m^2 has 10 prime factors, then m has 5.

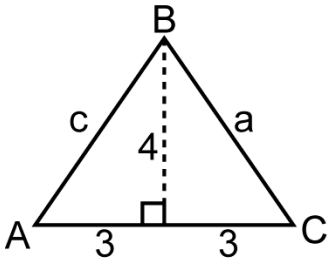
27. Look at the number line shown in the diagram. The two numbers that are twice as far from 2 as from -1 are 0 and -4.



28. The product of (x + 2) (x + 2) is $x^2 + 4x + 4$.

$$(x + 2)\ (x + 2) = x^2 + 4x + 4$$

29. If the base b is 6 cm and the area is 12 cm^2 , the height h is (6 x h ÷ 2 = 12) 4 cm. If the values of a and c are equal, we can find the perimeter of the triangle by finding the value of a. We know that $a^2 = 3^2 + 4^2$. We get a = 5 cm. The perimeter of the triangle is (6 + 5 + 5) 16 cm.



30. 300 = 2 x 2 x 3 x 5 x 5 = 2^2 x 3 x 5^2. The number of divisors of 300 is given by the product (2 + 1) x (1 + 1) x (2 + 1), which is 18.

31. The volume of the right cylinder is π x 2^2 x 9. The volume of the sphere is 4/3 π r^3 = 36 π. We find that $r^3 = 27$ and r = 3.

32. From x/3 = y/4, we get 4x = 3y and 8x = 6y. The value of (8x + 6y) : 2y is (6y + 6y) : 2y = 6.