# Mathematica Centrum

Together, let's shape the mathematicians of the future

PREPARATORY TEST 2009
COMPLETE SOLUTIONS

EULER (7th) – LAGRANGE (8th) – NEWTON (9th)

1. A quadrilateral is composed of 2 triangles. The average value of the angles in a quadrilateral is therefore 90° (360° ÷ 4).

2. In the product of powers of the same base, the exponents must be added. The value of n is $(2^4 \times 2^{11} = 2^{15})$ 15.

3. The value of $6^2 + 8^2$ is 100 or 10^2.

4. The number 51/24 is equal to 2 1/8 (51/24 = 17/8 = 2 1/8).

5. The value of (-18 + 6) ÷ 4 = -12 ÷ 4 = -3.

6. 1 km = 1 000 m.

7. Mathilda has written 18 consecutive integers in decreasing order. If the largest is 17, the 17th number that she has written is 1 and the 18th is 0.

8. Each dot in the diagram accounts for 2 glued faces. In all, there are (8 x 2) 16 faces that have glue on them.

9. After the first reflection (in the y-axis), the coordinates of the image of point A are (3,-2) and those of the image of B are (-3,5). After the second reflection (in the x-axis), the images of points A and B are, respectively, (3,2) and (-3,-5).

10. The measures of the 3 angles of a triangle are in the ratio 2 : 3 : 7. The value of the smallest angle is (180° ÷ 12 x 2) 30°.

11. A number is divided by 2 and the result is increased by 3; the new result is multiplied by 2 and gives a final answer of 20. Let's do the operations in reverse order keeping in mind that addition is the opposite of subtraction and multiplication is the opposite of division. The initial number ((20 ÷ 2 - 3) x 2) is equal to 14.
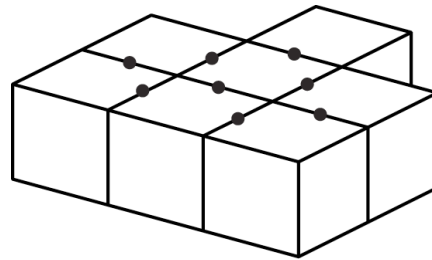
12. The number of small wheels that turn in a counter-clockwise direction is 4.
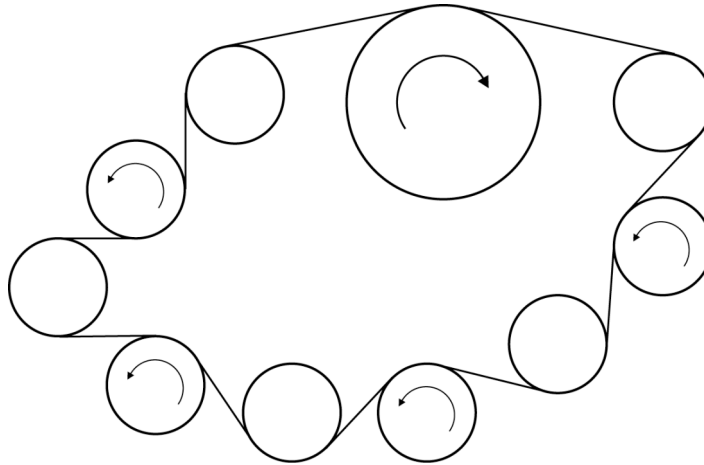
13. The 3 numbers, M, N, and P that will yield a minimum value for the expression M(N - P) are -20, 7, and -9. Substituting these 3 numbers in the expression, we get (-20(7 - -9) = -20 x 16) the minimum value of -320.

14. The date of the second Wednesday of the month could be the 1st, the 8th or the 27th. The values 1 and 27 are impossible. The value 8 is possible, because the Wednesday before (the first Wednesday of the month) would fall 7 days before, which would be the first. The first day of the month is a Wednesday.

15. The value of angle ABE is 2x. The value of 2x + 60 + x is equal to 180°, the value of x = 40°, and 2x = 80°.

16. The sum of 12 consecutive even integers is 12. These 12 numbers are -10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10, and 12. The largest of these numbers is 12 (don't forget that 0 is an even number).

**17.** If 2% of a number is equal to M, then 5% of this same number is equal to (M ÷ 2 x 5) 2.5 M.

18. Even numbers are multiples of 2. The even numbers that are multiples of 3 must be multiples (2 x 3) of 6. There are (6, 12, 18) 3 even natural numbers between 0 and 20 that are multiples of 3?

19. The average of 1/4 and 1/2 (1/4 + 1/2= 2/8 + 4/8 = 6/8) is equal to (6/8 ÷ 2) 3/8.

20. The reciprocal of 10 is equal to (1/10) 0.1.

21. Look at the diagram. The volume of the staircase is given by the expression 1 x 1 x 2 + 2 x 1 x 2. This expression is equal to (2 + 4) 6 m^3.

22. The expression -1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9 + 10 can be written as (-1 + 2) + (-3 + 4) + (-5 + 6) + (-7 + 8) + (-9 + 10) and is equal to 5.
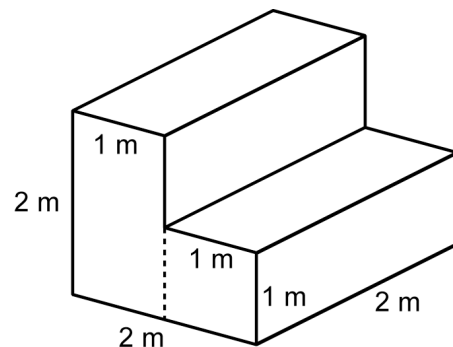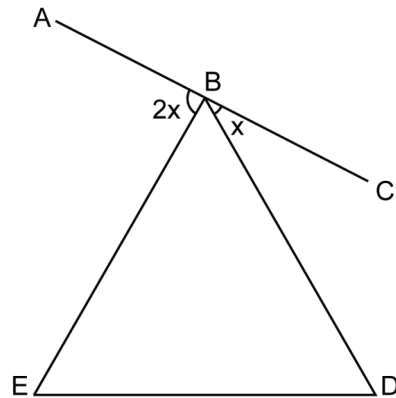
23. To calculate the value of the complex fraction, first we must calculate the value of the circled fraction and then continue to do the other operations (1 + 1/2) … until there are no more operations to be done. The operations are 1/(1 + 1) = 1/2, 1 + 1/2 = 3/2, 1 ÷ 3/2 = 2/3, 1 + 2/3 = 5/3. By the way, if this complex fraction were infinite, we would then have a fraction called a **continued fraction**. We would not be able to find its value by successive operations (there would be an infinite number of them). We can find the value of this continued fraction by using a short and very elegant mathematical method. The value of this **continued fraction** is exactly equal to the value of the golden number $(1 + \sqrt{5}) \div 2$, which is approximately equal to 1.62.

$$1 \quad + \quad \cfrac{1}{1 \quad + \quad \cfrac{1}{1 \quad + \quad 1}}$$

24. The number of zeros at the end of the product of 10! (1 x 2 x 3 x 4 x … x 10) is 2. We do not have to find the product of this long multiplication to know how many zeros appear at the end of it. Each zero is the result of the product of a 2 and a 5. There is only one 2 and one 5 that can generate a zero in the product of 10!. If we add this zero to the zero generated by the factor 10 (2 x 5), we get a total of 2.

25. The number 1 000 is divisible by 2, 4, 5, and 8. The number 960 is divisible by 2, 3, 4, 5, 6, and 8. Only 4 natural numbers between 1 and 10 (2, 4, 5, and 8) can divide both the numbers 960 and 1 000 without a remainder.

26. Mathilda prepares 120 g of a mixture of sand and water of which 90% is water. This initial mixture contains (0.9 x 120) 108 g of water. An hour later, she draws 20 g of water from the initial mixture (you can draw water from the mixture without drawing sand because sand and water form a heterogeneous mixture). The new mixture contains (108 g - 20 g) 88 g of water (don't forget that the final mixture contains 88 g of water and 12 g of sand). The percentage of water in the final mixture is (88 g ÷ 100 g) 88%.

27. The probability that he will get an even number is equal to the ratio of the sum of the angles that will yield an even number divided by 360°. The probability is therefore (100° ÷ 360°) 5/18.
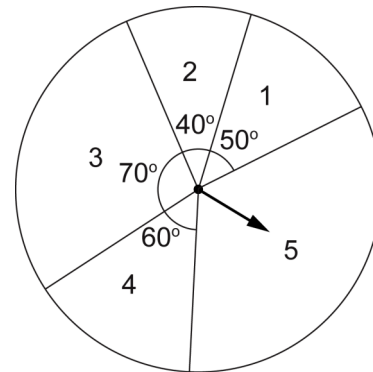
28. If x = 3, the value of x^2 - 2x + 6 is $(3)^2$ - 2(3) + 6 = 9.

29. For argument's sake, let's use a square with a side equal to 1. Its perimeter is 4. The perimeter of the circle is also 4. We can find the radius by using the equation 2 π r = 4. We find that r = 2/π. The area of the circle is π $(2/\pi)^2$. The area of the square is (1 x 1) 1. The area of the circle is (4/π ÷ 1) 4/π times greater than the area of the square.

30. The number of grids depends on the number of squares where we can place the centre of the central square of the block of 5 black squares. As shown in the diagram, the centre of the central square of the block can be placed in each square that contains a black dot. In all, there are (6 x 6) 36 squares with black dots and consequently 36 different crossword puzzle grids can be made from 8 x 8 grids of white squares.
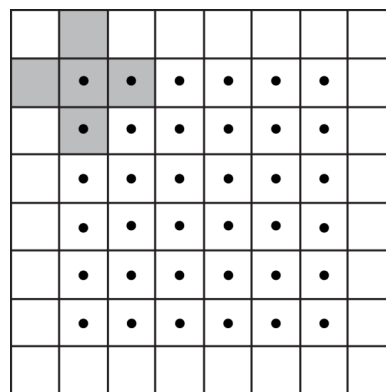
31. The small triangle is similar to the large one. We can write the proportion Y/3 = 2/4. This equation yields the value y = 3/2.

32. From n/3 = m/5, we find 5n = 3m. This equation is equivalent to 5n x 3 = 3m x 3, which becomes 15n = 9m. The value of (3m + 15n) is equal to (3m + 9m) 12m. Finally, from 12m ÷ 3m, we get a result of 4.